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**Unobserved Components in Economic Time Series**

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**ECONOMICS DEPARTMENT**

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**Unobserved Components in Economic Time Series**

**AGUSTÍN MARAVALL**

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# Unobserved Components in Economic Time Series

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## Abstract

The paper addresses the situation in which an economic variable, for which a series of observations is available, can be seen as the combination of several unobserved components (UC). UC models have been intensively used in applied economic research; they are often found, for example, in business cycle analysis. UC are also important in short-term policy and monitoring of economic variables, and an important example is seasonal adjustment. UC used in these two fields of applications (applied econometric research and statistical practical applications) often share the same basic structure. This paper deals with UC models displaying that type of structure. First, the limitations of ad-hoc fixed filters are briefly discussed; attention is focussed on the Hodrick-Prescott filter to detrend a series, and on the X11 filter to seasonally adjust a series. The paper develops then a general set-up for a model-based approach common to the vast majority of UC model applications. The basic feature is that the components follow linear stochastic processes. The problems of model identification, estimation and forecast of the components, diagnosis, and inference are sequentially addressed. The properties of the estimators (preliminary and historical) and of their associated estimation and forecasting errors are derived. Two examples are discussed: the quarterly series of US GNP (to illustrate business cycle analysis), and the monthly series of the UK money supply (to illustrate seasonal adjustment).

The paper contains some implications for applied econometric research. Two important ones are, first, that invertible models, such as AR or VAR models, cannot in general be used to model seasonally adjusted or detrended data. The second one is that to look at the business cycle in detrended series that are seasonally adjusted is a misleading procedure, since detrending plus seasonal adjustment will always induce a non-trivial spectral peak for a cyclical frequency.

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\*All the computations reported in the article are the output of a program "Signal Extraction in ARIMA Time Series" (in short, SEATS), described in Maravall and Gómez (1992), and available upon request. The program originated from one developed by J.P. Burman for seasonal adjustment at the Bank of England; to him I wish to express my gratitude. Thanks are also due to V. Gómez, G. Fiorentini, G. Caporello, and F. Canova.





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# Introduction

The paper addresses the situation in which an economic variable, for which a time series of observations is available, can be seen as the combination of several components. Having only observations on the aggregate variable, the analyst wishes to learn about the unobserved components and, in particular, about the joint distribution of their estimators and forecasts.

Unobserved components in time series have been of interest to economists for some time. A good review of the early developments and applications is contained in Nerlove, Grether and Carvalho (1979). The interest has developed along two separate (although related) fronts. First, unobserved component models are used in economic research in a variety of problems when a variable, supposed to play some relevant economic role, is not directly observable. For example, unobserved components have been used in modeling agents' reaction to (permanent or transitory) changes in the price level (Lucas, 1976), in analyzing the stability of some of the macroeconomic "big ratios" (Pagan, 1975), in relation to the "natural" level of the labor supply (Bull and Frydman, 1983), in modeling technical progress (Slade, 1989; Harvey and Marshall, 1991), in modeling credibility of the monetary authority (Weber, 1992), in measuring the persistence (or long-term effects) of economic shocks (Cochrane, 1988), or in estimating the underground economy (Aigner, Schneider and Ghosh, 1988).

Where, unquestionably, unobserved components have been most widely employed in economic research is in the area of macroeconomics. An important example is in the context of the Permanent Income Hypothesis, where permanent and transitory components play a central role (some references are Muth, 1960; Fama and French, 1988; Stock and Watson, 1988; Christiano and Eichenbaum, 1989; Quah, 1990). More generally, unobserved components are heavily used in the Business Cycle Literature, in the detection and analysis of the business cycle both at the methodological and applied level (see, for example, Sargent and Sims, 1977; Beveridge and Nelson, 1981; Nelson and Plosser, 1982; Kydland and Prescott, 1982; Harvey, 1985; Prescott, 1986; Watson, 1986; Clark, 1987; Crafts, Leybourne and Mills, 1989; Stock and Watson, 1988, 1991).

Most often, in the applications we have mentioned, a series is expressed as the sum of two components, such as a permanent and a transitory (or temporary) component, or as a trend and a cycle component. Typically, one of the components attempts to capture the trend-type nonstationarity of the series, while the other is a stationary component. Often, moreover, the series to be decomposed has been previously seasonally adjusted, and hence a third component containing the seasonal variation is also implicit in the decomposition.

But, besides their use in applied econometric research, unobserved components play an important role in short-term economic policy making and monitoring of the economy. Typically, short-term policy and evaluation is based on the seasonally adjusted series. (It is indeed the case that, for many important macroeconomic series, seasonal movements

dominate the short-term evolution of the series.) When the seasonally adjusted series displays undesirable erraticity, some further smoothing may be performed, attempting to capture the trend component. In fact, the standard decomposition of macroeconomic variables, as used in applied institutions or agencies, can be seen as decomposing the series into a nonstationary trend, a nonstationary seasonal, and a stationary irregular component (see, for example, Moore et al., 1981). This practical need for unobserved components estimation has also generated a large amount of research, most of it developed in the statistics field (see, for example, Zellner, 1978, 1983, and the references in Den Butter and Fase, 1991).

Unobserved components used in these two fields of applications (applied econometric research and statistical practical applications) often share the same basic structure. This paper deals with unobserved component models displaying that type of structure. First, linear filters are introduced and ad-hoc fixed filters are briefly discussed. Attention is focussed on the Hodrick-Prescott filter to detrend a series, and on the X11 filter to seasonally adjust a series. Some dangers of ad-hoc filtering are illustrated, in particular the risk of spurious results and the effects of over and underestimation of a component. Some limitations are also pointed out, such as its incapacity to provide estimation standard errors and forecasts (with their associated standard errors). This is discussed in sections 1 and 2.

Section 3 presents the general set-up for a model-based approach common to the vast majority of unobserved component model applications. The basic feature is that the components follow linear stochastic processes. Section 4 reviews and compares some of the most frequent specifications for the most common components (such as the trend, cyclical, seasonal, transitory or irregular components). For a given series, the lack of a unique decomposition reflects a general underidentification problem inherent in unobserved component models. This problem is addressed in section 5; the most relevant solutions are discussed and the implied decompositions compared. Section 6 illustrates the presentation with an example, the quarterly series of US GNP, which has been the center of attention in business cycle research.

Estimation of the components is considered in section 7. The optimal estimator is the conditional expectation of the component given the observed series. Since it is well suited for analytical discussion, the Wiener-Kolmogorov representation of that conditional expectation is considered. First, we present the case of an infinite realization of the series and look at the component estimation filter; for most applied cases, it will be close to the one that yields the estimator of the component for the central years of the series (the historical estimator), independently of whether it has been obtained with the Wiener-Kolmogorov or with the Kalman filter.

Section 8 analyses the properties of the estimator. It is seen generated by a linear stochastic process, structurally different from the process assumed to generate the component. This difference is analysed in terms of the direction of the bias, the auto and crosscorrelation structure, and the corresponding spectra. An application to diagnostic checking of the model, and some estimation extensions are also considered. Two



important implications for applied econometric research are discussed at the end of the section. One concerns the use of seasonally adjusted series (also of other components such as the trend or the detrended series) in econometric testing and model building, as well as in some commonly used unobserved component models. The second one concerns the procedure of looking for business cycles in a detrended series which has been seasonally adjusted.

Estimation and forecasting of unobserved components for a finite sample, as well as preliminary estimators are considered in section 9. Analytical expressions are derived and the structure of the preliminary estimator and associated revisions is discussed. Some illustrations are provided in the field of applied econometrics and in the field of statistical practical applications.

Section 10 deals with estimation errors. The models generating the errors in the historical and preliminary estimators are presented. Since the errors depend on the model specification, some relevant implications concerning identification of the unobserved component model are derived. Finally, the use of an unobserved component model in inference is illustrated with the monthly series of the U.K. money supply and the standard procedures used in monitoring monetary aggregate series.

## 1 Linear Filters

Consider an observed times series  $x_t$  which we wish to express as

$$x_t = \sum_{i=1}^k x_{it}, \quad (1.1)$$

where  $x_{1t}, \dots, x_{kt}$  denote  $k$  unobserved components. Since often interest centers on one of the components, it will prove useful to rewrite (1.1) as the sum of two components:

$$x_t = m_t + n_t, \quad (1.2)$$

where  $m$  denotes the component of interest (the "signal"), and  $n$  denotes the sum of the other components (the non-signal or "noise").

For components such as a trend or a seasonal component, deterministic specifications, such as fixed polynomials in time or cosine functions, have been employed, and references can be found in Stephenson and Farr (1972), Fuller (1976), and Hylleberg (1986). The gradual realization that economic time series display moving or evolving trend and seasonal behavior lead to the replacement of deterministic models by the so-called Moving Average methods, which can be seen as approximating the trend by local polynomials (see Kendall, 1976) and the seasonal by local cosine functions (see Box, Hillmer and Tiao, 1978).

The most widely used moving average filters are linear (except for some possible tapering of outliers) and, for the observations not close to either end of the series, centered and symmetric. This last property is due to the requirement that the filter induce

no phase-shift in the estimation of the component; since it is most desirable that the underlying seasonal or cyclical ups and downs of the series be properly timed, the requirement seems a sensible one (a good presentation of moving average filters can be found in Gourieroux and Monfort, 1990). If  $B$  denotes the lag operator and  $F = B^{-1}$  denotes the forward operator, such that  $B^K x_t = x_{t-K}$  and  $F^K x_t = x_{t+K}$ , a linear symmetric moving average filter is of the form

$$\hat{x}_{it} = C_i(B) x_t, \quad (1.3)$$

$$C_i(B) = c_0 + \sum_{i=1}^r c_i (B^i + F^i). \quad (1.4)$$

A filter for a trend component will naturally be designed to capture the series variation associated with the long-term movements (i.e., the movements displaying very low frequencies), and the seasonal component filter will be constructed to capture variability associated with seasonal frequencies. Since the components are often associated with specific frequencies, the frequency domain view will be of help in analysing the properties of the filters. Broadly, if a component is designed to capture the series variation for a specific frequency region, the moving average filter to estimate the component can be seen as a bandpass filter, that should have a close to 1 gain in that region, and a zero gain for other frequencies. Filters have been constructed in an ad-hoc manner to display that bandpass structure. These filters are fixed (perhaps allowing for a few options), and independent of the time series under analysis. One important example in the area of applied economic research, where interest centers on detrending of series, is the Hodrick-Prescott (HP) filter; see Hodrick and Prescott (1980) and Prescott (1986). In the area of data treatment for policy and monitoring of the economy, where seasonal adjustment is the most frequent application, massive use is made of X11-type filters; see Shiskin et al. (1967) and Dagum (1980). (Linear expression for the central X11 filter can be found in Ghysels and Perron, 1993.)

Let  $\omega$  denote frequency, measured in radians; the frequency domain representation of the HP filter (for the recommended value of  $\lambda = 1600$ ) and of the symmetric X11 quarterly filter are displayed in figures 1 and 2, which evidence the bandpass character of the two filters. The HP filter has a value of 1 for frequencies near to 0, and a value of 0 for frequencies associated with periods of less than 4 or 5 years. The X11 filter removes the variance in the neighborhood of  $\omega = \pi$  and  $\omega = \pi/2$ , the once- and twice-a-year seasonal frequencies.

An important property of two-sided symmetric filters of the type (1.4) is that the estimator of the component  $x_{it}$  cannot be estimated with (1.3) when  $t$  is close enough to either end of the series. If  $[x_t] = [x_1, x_2, \dots, x_T]$  denotes the observed series, when  $t < r$ , unavailable starting values of  $x$  are required; when  $t > T - r$ , future observations are needed to complete the filter. Ad-hoc filters (except for X11 ARIMA) truncate the filter for those end observations with ad-hoc weights. Therefore, the centered and symmetric filter characterizes "historical" or final estimators. For recent enough periods, in particular, asymmetric filters have to be used, which yield preliminary estimators. As



time passes and new observations become available, those preliminary estimators will be revised until the final estimator is eventually obtained. To this issue I shall come back later.

It would seem that a convenient feature of ad-hoc filters could be that, by defining the component as the outcome of the filter (see Prescott, 1986), the issue of defining the component properly is simplified. This simplification, however, is misleading. To illustrate the difficulties involved in using this definition, consider an example: the trend component obtained with the HP filter applied to the series of US GNP. The series is discussed in detail in section 6; it consists of 35 years of quarterly observations. Assume we are in the middle of year 18 ( $t = 70$ ), and use the HP filter to estimate the trend for that period. According to the definition of Prescott, this estimator is the trend for  $t = 70$ . But if one more quarter is observed, the HP filter yields a different estimator for  $t = 70$ . Additional quarters will further change the estimator, and the HP filter is in fact a filter that implies a very long revision period. Figure 3 displays the trend estimated for  $t = 70$  as the length of the series increases from 70 to 140 observations. The estimator is seen to fluctuate considerably, and takes nearly 10 years to converge. Which of these estimators is the trend? Obviously, a preliminary estimator is inadequate, since one would then conclude that new information deteriorates the estimator. If the trend is defined as the historical estimator, then it will take 10 years into the future to know today's trend.

More relevant virtues of ad-hoc filters are that they are simple and easy to use. This is an important property when there is an actual need of estimating components for a large number of series. Thus one can understand that in a statistical agency, having to routinely seasonally adjust hundreds of series, heavy use is made of an ad-hoc filter such as X11. In applied economic research, where attention centers on methodological issues having to do with few series, the convenience of using ad-hoc filters is far less clear.

## 2 Ad-Hoc Filtering: Dangers and Limitations

The dangers and limitations of ad-hoc filtering have been often pointed out. Here we illustrate with simple examples some of the most important ones.

(A) The danger of spurious adjustment is illustrated in figures 4 and 5. For a white-noise series with unit variance (expressed, for convenience, in units of  $2\pi$ ), the HP filter yields a detrended series with spectrum that of figure 4. The spectrum can be seen as a very wide peak for  $\omega = \pi$ , and hence the detrended series will behave as a (noninvertible) strongly stochastic component with a period of 2 (quarters). Yet, by construction, the detrended series should be the white-noise input. For the same type of input, X11 extracts a seasonal component with spectrum that of figure 5. The spectrum is certainly that of a seasonal component, but it is spurious since the white-noise series contained no seasonality.

For a white-noise series, it is obvious that the filter that seasonally adjusts the series should simply be 1. On the other hand, if the series under analysis has a spectrum like that of figure 5, the filter to seasonally adjust the series should be 0, since all variation is seasonal. The filter should depend, thus, on the characteristics of the series. To illustrate this dependence, consider the model

$$\nabla \nabla_4 x_t = (1 - \theta_1 B) (1 - \theta_4 B^4) a_t, \quad (2.1)$$

where  $a_t$  is a white-noise innovation (with variance  $V_a$ ),  $\nabla = 1 - B$ ,  $\nabla_4 = 1 - B^4$ , and the two moving average parameters lie between  $-1$  and  $1$ . It is a model similar to the so-called Airline Model of Box and Jenkins (1970, chap. 9), for quarterly series. On the one hand, it is often encountered in practice; on the other hand, it provides an excellent reference example. The model accepts a sensible decomposition into trend, seasonal, and irregular components (see Hillmer and Tiao, 1982). As  $\theta_1$  approaches 1, model (2.1) tends towards the model

$$\nabla_4 x_t = (1 - \theta_4 B^4) a_t + \mu_o,$$

with a more deterministic trend. (Notice that, since  $\nabla_4$  contains the root  $(1 - B)$ ,  $\mu_o$  is the — now deterministic — slope of the trend.) Similarly, when  $\theta_4$  becomes 1, the seasonality in (2.1) becomes deterministic. Thus, broadly, the parameters  $\theta_1$  and  $\theta_4$  can be interpreted as a measure of how close to deterministic the trend and the seasonal components, respectively, are. The behavior of a component is easily illustrated in the frequency domain. Model (2.1) does not have a proper spectrum, since it is nonstationary; it will be useful however to use its pseudo-spectrum (see Hatanaka, 1967, or Harvey, 1989). For a linear model  $x_t = \Psi(B) a_t$ , with  $\Psi(B) = \theta(B)/\phi(B)$ , where  $\theta(B)$  and  $\phi(B)$  are finite polynomials in  $B$ , the pseudo-spectrum is the Fourier transform of  $\Psi(B)\Psi(F)V_a$ ; it will display infinite peaks for the frequencies associated with the unit roots of  $\phi(B)$ . In what follows, the term spectrum will be used also to denote a pseudospectrum.

The closer to deterministic behavior of a component is revealed by the width of the spectral peak for the relevant frequency. Thus, for example, figure 6 displays the spectra of two series both following models of the type (2.1) with  $V_a = 1$ , one with  $\theta_1 = -.1$ ,  $\theta_4 = .7$ , and the other with  $\theta_1 = .7$ ,  $\theta_4 = -.1$ . Comparing the two spectra, the one with the continuous line contains a more stable (closer to deterministic) trend, and a more unstable seasonal. Since an ad-hoc filter displays holes of a fixed width, the filter will underadjust when the width of the spectral peak in the series is larger than the width of the filter hole. Alternatively, it will overadjust when the spectral peak in the series is narrower than that for which the filter has been designed. Figure 7 illustrates the effect of using X11 on the series with unstable seasonality of figure 6. In part (a) it is seen how the width of the filter is narrower than the spectral peak in the series. Part (b) shows how the underestimation of the seasonal component has spillover effects, reflected by peaks in the seasonally adjusted series spectrum in the vicinity of the seasonal frequencies. For the case of the HP filter, figure 8 illustrates its application to the series with a relatively unstable trend of figure 6. (Since the HP trend filter becomes



zero much before the first seasonal frequency, it will have no effect on the spectrum for the seasonal frequencies.) As seen in part (a) of the figure, the filter underestimates the peak around the zero frequency contained in the series, and part (b) shows the effect of this underestimation: the detrended series will be overestimated, and will exhibit a strong cycle, induced entirely by the underestimation of the trend. The spurious cycle is associated with a period of approximate 6 years. (It is worth mentioning that the quarterly Airline Model used to illustrate the danger of underestimation with the HP filter is in fact very close to the model appropriate for the US GNP series, which has been the center of attention in business cycle research; see section 6.) The danger of spurious results induced by the HP filter have been often pointed out; examples are found in King and Rebelo (1993), Cogley (1990), Canova (1991), and Harvey and Jaeger (1991).

(B) The lack of a proper statistical model limits in many important ways the usefulness of ad-hoc filters. First, it makes it difficult to detect the cases in which the filter is not appropriate for the series at hand. Moreover, if such is the case, there is no systematic procedure to overcome the filter inadequacies. Second, even when appropriate, ad-hoc filtering does not provide the basis for rigorous inference. Given that the filter yields an estimator of the unobserved component, it would be desirable to know the properties of the estimator, and in particular the underlying estimation errors; as shall be seen later, this knowledge may have relevant policy implications. Further, ad-hoc filters do not provide the basis for obtaining forecasts of the components, which can also be of interest. Forecasts and estimation (and forecasting) errors of the components will be discussed in sections 9 and 10.

### 3 The Model-Based Approach

To overcome the black-box character of ad-hoc filtering, and the limitations mentioned in the previous section, over the last 15 years, new approaches to unobserved component estimation, based on parametric models, have been developed. These models are closely related to the Autoregressive (AR), Integrated (I) Moving Average (MA) — or ARIMA — models, popularized by Box and Jenkins (1970). They have been the subject of considerable statistical research having to do with practical applications, such as seasonal adjustment. They have also been used intensively in applied econometric research and, in fact, most of the references given in the Introduction contain model-based applications.

Except for some nonlinear extensions (examples are Carlin and Dempster, 1989; Hamilton, 1989; Harvey, Ruiz and Sentana, 1992), the vast majority of model-based approaches use a linear assumption, which I shall state as follows:

**Assumption 1:** Each component in expression (1.1) can be seen as the outcome of a linear stochastic process, of the type

$$\delta_i(B) x_{it} = \psi_i(B) a_{it}, \quad (3.1)$$



where  $a_{it}$  denotes a white-noise variable and the polynomial  $\psi_i(B)$  can be expressed as  $\psi_i(B) = \theta_i(B)/\varphi_i(B)$ . The polynomials  $\delta_i(B)$ ,  $\theta_i(B)$ , and  $\varphi_i(B)$  are of finite order. The roots of  $\delta_i(B)$  are on the unit circle; those of  $\varphi_i(B)$  lie outside and, finally,  $\theta_i(B)$  has all roots on or outside the unit circle. For each  $i$ , the three polynomials are prime. ■

Throughout the paper, a white-noise variable will denote a zero-mean, normally, identically, and independently distributed variable. I shall refer to  $a_{it}$  as the  $p$ (seudo)-innovation associated with component  $i$ . The variable  $z_{it} = \delta_i(B)x_t$  represents the stationary transformation of  $x_t$ , and the parametric expression for the component will be

$$\varphi_i(B) \delta_i(B) x_{it} = \theta_i(B) a_{it}, \quad (3.2)$$

or, in compact form,

$$\phi_i(B) x_{it} = \theta_i(B) a_{it}, \quad (3.3)$$

where  $\phi_i(B)$  is the product of the stationary and the nonstationary AR polynomials. The orders of the polynomials  $\phi_i(B)$  and  $\theta_i(B)$  are  $p_i$  and  $q_i$ , respectively. Since different roots of the AR polynomial induce peaks in the spectrum of the series for different frequencies, and given that different components are associated with spectral peaks for different frequencies, the following assumption will be made.

**Assumption 2:** The polynomials  $\phi_i(B)$  and  $\phi_j(B)$ ,  $i \neq j$ , share no common root. ■

From (1.1) and (3.3),

$$x_t = \sum_{i=1}^k \frac{\theta_i(B)}{\phi_i(B)} a_{it},$$

which implies, under Assumption 2, that  $x_t$  is also the outcome of a linear process:

$$\varphi(B) \delta(B) x_t = \theta(B) a_t, \quad (3.4)$$

where  $\delta(B) = \prod_{i=1}^k \delta_i(B)$ ,  $\varphi(B) = \prod_{i=1}^k \varphi_i(B)$ ,  $\theta(B)$  is a polynomial of finite order in  $B$  (say, of order  $q$ ), and  $a_t$  is a white-noise variable. Expression (3.4) can be rewritten as

$$\phi(B) x_t = \theta(B) a_t, \quad (3.5)$$

and consistency between the overall model and the ones for the components implies the two constraints:

$$\phi(B) = \prod_{i=1}^k \phi_i(B), \quad (3.6)$$

$$\theta(B) a_t = \sum_{i=1}^k \theta_i(B) \phi_{ni}(B) a_{it}, \quad (3.7)$$

where  $\phi_{ni}(B)$  is the product of all AR polynomials excluding  $\phi_i(B)$ , that is

$$\phi_{ni}(B) = \prod_{j=1(j \neq i)}^k \phi_j(B).$$

Assumption 1 allows for noninvertible components. I shall require, however, the model for the observed series to be invertible. Since noninvertibility is associated with a spectral zero, there should be no frequency for which all component spectra become zero. This is implied by the following assumption.

**Assumption 3:** The polynomials  $\theta_i(B)$ ,  $i = 1, \dots, k$ , share no unit root in common.

## 4 Characterization of the Components

In the same way that there is no universally accepted definition of a trend or of a seasonal component, there is no universally accepted model specification for a particular component. When building the overall model, one can proceed by directly specifying a model for each unobserved component that in some way captures the prior beliefs about the component. This is the so-called Structural Time Series (STS) approach, and some basic references are Engle (1978), Gersch and Kitagawa (1983), Harvey and Todd (1983), and Harvey (1989); direct specification of the component model is also the most used approach in applied econometrics. Alternatively, since observations are only available on the overall series, one can proceed by identifying first a model for the observed series, and then deriving appropriate models for the components that are compatible with the "observed" one. This is the so-called ARIMA Model Based (AMB) approach, and basic references are Box, Hillmer and Tiao (1978), Burman (1980), Hillmer and Tiao (1982), and Bell and Hillmer (1984). It is of interest to review some of the most common specifications used to characterize some of the most common components.

### a) Trend Component

Consider the deterministic linear trend  $m_t = a + \mu t$ , for which

$$\nabla m_t = \mu, \quad (4.1)$$

$$\nabla^2 m_t = 0. \quad (4.2)$$

A moving or stochastic trend will not exactly satisfy (4.1) and (4.2) at every period; instead, one can assume that for a stochastic trend, the above relationships are perturbed every period by a random shock, with a zero mean and a small variance. When the shock is a white-noise variable, (4.1) becomes

$$\nabla m_t = \mu + a_{mt}, \quad (4.3)$$

where  $a_{mt}$  is the white-noise shock. Its variance  $V_m$  will reflect how important is the random element in the trend. Model (4.3) is the standard "random walk plus drift" specification, widely used in econometric applications (see, for example, Stock and Watson, 1988). Alternatively, (4.2) can generate the stochastic trend model

$$\nabla^2 m_t = a_{mt}, \quad (4.4)$$



similar to the one in Gersch and Kitagawa (1983). The STS approach of Harvey and Todd (1983) models the trend as a random walk plus drift process,

$$\nabla m_t = \mu_t + a_{mt}, \quad (4.5.a)$$

where the drift is also generated by a random walk, as in

$$\nabla \mu_t = a_{\mu t}, \quad (4.5.b)$$

where  $a_{\mu t}$  is white noise. (Similar types of “second-order” random walks are also found in the trend models of Harrison and Stevens, 1976, and of Ng and Young, 1990.) Writing (4.5) as  $\nabla^2 m_t = a_{\mu t} + \nabla a_{mt}$ , it is seen to be equivalent to a model of the type

$$\nabla^2 m_t = (1 - \theta_m B) a_{mt}, \quad (4.6)$$

with the constraint  $\theta_m > 0$ . Notice that as  $\theta_m$  approaches 1, model (4.6) tends to the standard random walk plus drift model (4.3). Since  $\mu$  is the slope of the linear trend, the choice between an I(1) and an I(2) trend reflects the choice between a constant and a time-varying slope. Finally, in the AMB approach, the model for the trend will depend on the model for the observed series and, in particular, its order of integration at  $\omega = 0$  will be the same. As an example for the quarterly Airline Model (2.1), the trend follows an IMA(2, 2) model.

In general the model for the stochastic trend is of the form

$$\nabla^d m_t = \psi_m(B) a_{mt}, \quad (4.7)$$

with  $d = 1$  or  $2$ , and  $\psi_m(B) a_{mt}$  a low-order ARMA process. The same type of stochastic linear trend specification is often used to model economic variables that are treated as unobserved components. Some examples are the model for the permanent component in permanent/transitory-type of decompositions (Muth, 1960; Pagan, 1975; Clark, 1987; Stock and Watson, 1988; and Quah, 1990); the model for the unobserved planned policy targets (Weber, 1992); for technical progress (Slade, 1989); for productivity effects (Harvey, et al., 1986); or for the general “state of the economy” in Stock and Watson (1989). (A more complete discussion of stochastic linear trends is contained in Maravall, 1993a.)

#### b) Seasonal Component and Seasonally Adjusted Series

In most economic applications of unobserved components, the seasonal component is not explicitly dealt with. If the series contains seasonality, as many macroeconomic series do, the seasonally adjusted data is typically employed. Although not explicitly modeled, the seasonal component may certainly affect the results of the analysis, and some limitations associated with the use of seasonally adjusted data have been pointed out (see, for example, Wallis, 1974; Ghysels and Perron, 1993; Miron, 1986; and Osborn, 1988). As shall be seen later, moreover, the seasonally adjusted series are likely to be particularly inadequate for business cycle analysis. Since, within the model-based



framework, it can be done in a straightforward manner, it is preferable to incorporate the seasonal component as part of the model to be estimated, jointly with the rest of the components. In the statistical applications having to do with economic policy or monitoring, explicitly modeling the seasonal component is important, since seasonal adjustment is the most common application.

The structure of the model for the stochastic seasonal component can be motivated in a manner similar to that used for the trend. Let  $s$  denote the number of observations per year, and  $m_t$  a deterministic seasonal component (expressed as the sum of dummy variables or of cosine functions). Then the sum of 12 consecutive seasonal components will exactly cancel out, that is

$$S m_t = 0, \quad (4.8)$$

where  $S = 1 + B + \dots + B^{s-1}$ . If the component is moving in time, (4.8) cannot be expected to be satisfied at each period, although the deviation should average out and be relatively small. If we assume that equation (4.8) is subject each period to a random shock, a stochastic model for the seasonal component is obtained. If the shock, for example, is the white-noise variable  $a_{mt}$ , the model becomes

$$S m_t = a_{mt}, \quad (4.9)$$

which is the model for the seasonal component in the STS approach of Harvey and Todd (1983), and also the seasonal model specification used in the approach of Gersch and Kitagawa (1983). There is no compelling reason for the deviations from zero in  $S m_t$  to be uncorrelated and, for example, in the AMB approach, the seasonal component obtained in the decomposition of (2.1) is of the form

$$S m_t = \theta_m(B) a_{mt}, \quad (4.10)$$

where  $\theta_m(B)$  is of order 3. Similar types of models can be found in, for example, Aoki (1990), and Kohn and Ansley (1987). More generally, expression (3.1), with  $\delta_s(B) = S$  and  $\psi_s(B)$  a relatively low-order ARMA process, is indeed a frequent specification for the seasonal component.

Some departures are found in Burrige and Wallis (1984), where  $\theta_s(B)$  is of a relatively high order (although parsimonious), or in Hylleberg et al. (1990), where some seasonal harmonics are allowed not to be present, and hence some of the unit roots in  $S$  may be missing. In many of the earlier model-based approaches, the seasonal component was modeled as having  $\nabla_s$  in its AR part. In the presence of a trend, this specification would be ruled out by Assumption 2. In fact,  $\nabla_s$  includes the root  $(1 - B)$ , which should not be a part of the seasonal component, otherwise the filter that yields the seasonal estimator would contain part of the trend. (A more complete discussion of the seasonal component model specification is contained in Maravall, 1989.)

As for the seasonally adjusted series (also an unobserved component), its structure will depend on which components, other than the seasonal, are present in the series. For example, in the AMB decomposition of model (2.1), the seasonally adjusted series

equals the sum of an IMA(2, 2) trend and a white-noise irregular. Thus the adjusted series will also follow an IMA(2, 2) model. In the STS decomposition of Harvey and Todd (1983), when no cycle is present, the adjusted series (the sum of an IMA(2, 1) trend and a white-noise irregular) also follows an IMA(2, 2) model.

### c) Cyclical Component

There have been two different ways of characterizing the cyclical component. One has been as a periodic stochastic component, which can be rationalized as in the two previous cases: It is well known that, for  $\phi_1^2 < 4\phi_2$ , the difference equation

$$x_t + \phi_1 x_{t-1} + \phi_2 x_{t-2} = 0 \quad (4.11)$$

displays deterministic periodic behavior of the type  $x_t = A_0 r^t \cos(\omega t + A_1)$ , where  $r = \sqrt{\phi_2}$  is the modulus and  $\omega = \arccos[-\phi_1/2\sqrt{\phi_2}]$  is the frequency (in radians). For some values of  $\phi_1$  and  $\phi_2$ ,  $\omega$  will fall in the interval  $0 < \omega < \omega_1$ , where  $\omega_1$  is the fundamental seasonal frequency  $\omega_1 = 2\pi/s$ , with period equal to  $s$ . Values of  $\omega$  inside that interval will generate deterministic cycles of period longer than a year. As before, if the cyclical component is of the moving type, (4.11) will not be satisfied exactly. If it is assumed that the deviations from zero are white noise, the linear stochastic model for the cycle becomes an AR(2) model; more generally, allowing for some autocorrelation in the deviations, the model for the cycle can be written as:

$$m_t + \phi_1 m_{t-1} + \phi_2 m_{t-2} = \theta_m(B) a_{mt}, \quad (4.12)$$

where  $\theta_m(B)$  is a low-order MA polynomial. Models of this type will be referred to as "periodic cycles", and they have been used in many applications (examples are Jenkins, 1979; Kitchell and Peña, 1984; Harvey, 1985; Crafts, Leybourne and Mills, 1989).

In macroeconomics, however, the cycle is seldom seen as a periodic behavior in the previous sense. Typically, the cycle represents the deviations (that are not seasonal) from a long-term component or trend. The cycle is therefore measured as the residuals obtained after detrending a seasonally adjusted series. Of course, these residuals need not exhibit periodic cyclical behavior, and may follow, in general, a stationary ARMA process. What characterizes this concept of the cycle is that it represents in some sense the stationary variations of the series. I shall refer to this view of the cycle as the "business cycle" approach.

Within the model-based approach, a different concept of the cycle has been recently proposed by Stock and Watson (1989, 1991, 1993). The cycle is given by sequences of growth of an unobserved component (the state of the economy) above or below some threshold. The unobserved component is then modeled as a stochastic trend.

### d) Irregular and Transitory Component

In statistical decompositions of economic time series, the series is often expressed as the sum of a trend, a seasonal, and an irregular component. In practice, the irregular component is obtained as the residual after the trend and seasonality have been removed.



In the model-based approach, the irregular component is a stationary low-order ARMA process, quite frequently simply white noise.

Transitory (or temporary) components are used in econometrics to capture short-term variability of the series and are equal to the series minus its permanent component. The permanent component, as already mentioned, is typically modelled as a stochastic trend, and hence the transitory component can also be seen as the detrended (often seasonally adjusted) series. Again, the transitory component will be a stationary ARMA process; in econometrics, it is frequently modeled as a finite AR process.

e) A Remark on Stationarity and Model Specification

Nonstationarity of the trend and of the seasonal component is somewhat implied by the very nature of the component. If the trend is stationary, in which way can it measure the long-term evolution of the series? As for the seasonal component, the basic requirement that its sum over a year span should, on average, be zero, implies the presence of the operator  $S$  in the AR expression for the component, as in (4.10), and hence the presence of nonstationary seasonality. For the case of the periodic cyclical component, if  $\phi_2 = 1$  the component will be nonstationary. In practice, however, most cycles detected using models of the type (4.12) are found to be stationary.

The business cycle, the irregular, and the transitory components are modelled as stationary processes and, basically, can be seen as the residual obtained after the trend and seasonal components have been removed from the series. This common basic structure does not imply that, for the same series, the three components have to follow necessarily the same model. A simple example will illustrate the point:

Let  $x_t$  be a (nonseasonal) series, the output of the process  $(1 - .7B) \nabla x_t = \theta(B) a_t$ , where  $\theta(B)$  is a low-order MA. If one is interested in short-term analysis (as is the case in statistical practical applications) and wishes to remove from  $x_t$  only white-noise variation, one may consider the model  $x_t = m_t + n_t$ , where

$$\begin{aligned}(1 - .7B) \nabla m_t &= a_{mt} \\ n_t &= a_{nt}.\end{aligned}$$

On the contrary, if interest centers in long-run analysis (as in some econometric applications), one may prefer an alternative specification of the type

$$\begin{aligned}\nabla m_t &= a_{mt} \\ (1 - .7B) n_t &= a_{nt}.\end{aligned}$$

Both specifications are perfectly reasonable; the second one will yield a smoother stationary component. In fact, it is a virtue of the model-based approach that the purpose of the analysis can be incorporated in the specification of the models.



## 5 Identification

### a) The General Problem

Conditional on some starting conditions, and under Assumptions 1–3, the overall ARIMA expression (3.5) determines entirely the joint distribution of the observations (or of the transformation  $\delta(B)x_t$ ). Since the parameters of (3.5) can be estimated consistently, and given that our interest centers on estimation of the unobserved components, for most of the remaining discussion I shall make the following assumption:

**Assumption 4:** The polynomials  $\phi(B)$  and  $\theta(B)$ , as well as the variance of  $a_t(V_a)$  in model (3.5) are known. ■

Considering expression (3.6) and Assumption 2, factorization of  $\phi(B)$  directly yields the polynomials  $\phi_i(B)$  of the unobserved components. The different roots may be allocated to the different components according to the behavior they induce in the series. Thus, the AR polynomials of the components are identified and can be obtained from the AR polynomial in the model for the observed series. The parameters that remain to be determined are those in the MA polynomials  $\theta_i(B)$ ,  $i = 1, \dots, k$ , and in the contemporaneous covariance matrix of the vector of  $p$ -innovations, namely those in  $\Sigma = [\text{cov}(a_{it}, a_{jt})]$ . These parameters have to be obtained from the identity (3.7). Under the Normality assumption, if the system of equations that results from equating the autocovariances of the left-hand side (l.h.s.) to the autocovariances of the right-hand side (r.h.s.) of the identity (3.7) has a locally isolated solution for the parameters in  $\theta_i(B)$  and  $\Sigma$ , the models for the components are identified. Obviously, without any additional assumption there will be an infinite number of possible specifications that will satisfy (3.7) (and the implied system of covariance equations). In order to isolate a particular solution (i.e., in order to reach identification) additional restrictions are needed.

### b) Restrictions on the Covariance Matrix $\Sigma$

On occasion, the components are allowed to be correlated; see, for example, Watson (1986) and Ghysels (1987). The most widely used decomposition that allows for correlated components is the one proposed by Beveridge and Nelson (1981). If  $x_t$  denotes an I(1) variable such that  $\nabla x_t$  has the Wold representation  $\nabla x_t = \Psi(B)a_t$ , then  $x_t$  can always be expressed as the sum of a permanent and a transitory component, where the permanent component is given by  $\nabla m_t = \Psi(1)a_t$ , and the transitory component is equal to  $n_t = \Psi^*(B)a_t$ , where  $\Psi^*(B)$  satisfies  $(1 - B)\Psi^*(B) = \Psi(B) - \Psi(1)$ . The Beveridge–Nelson decomposition can be seen as an ingenious decomposition of an I(1) variable, but it does not properly fit into the unobserved components framework, since the components are, in fact, observable. This is easily seen by rewriting, for example,  $m_t$  as  $m_t = \Psi(1)\Psi(B)^{-1}x_t$ , and hence both components are defined as linear combinations of the observed series. The assumption, besides, that the permanent and transitory component share, at every period, the same innovation is a strong assumption, of limited appeal. Instead, I shall assume that what causes the underlying long-term evolution of an economic variable is different from what causes it to display seasonal variation, and

from what causes the transitory deviations. Accordingly, the following constraint will be imposed.

**Assumption 5:** The  $p$ -innovations  $a_{it}$  and  $a_{jt}$  are uncorrelated for  $i \neq j$ . That is,

$$\Sigma = \text{diag}(V_i). \blacksquare$$

Assumption 5 is a standard assumption in statistical practical applications. The choice between correlated and orthogonal  $p$ -innovations, as shall be seen in section 8, is less drastic than it may appear. Ultimately, the component estimators will be linear projections on the observed times series  $x_t$ , and hence linear filters of the innovations  $a_t$ . In fact, the Beveridge–Nelson decomposition can be seen as the estimators that are obtained for a particular permanent/transitory decomposition with uncorrelated components (Watson, 1986).

### c) Additional Restrictions

Assumption 5 is not enough to identify the model, and more restrictions are needed. The discussion will be clearer if we look at a particular example, namely the quarterly Airline Model (2.1). Assume we wish to decompose  $x_t$  into a trend ( $x_{mt}$ ), a seasonal ( $x_{st}$ ), and an irregular component  $x_{ut}$ , as in (1.1). Since the AR part of (2.1) can be rewritten  $\nabla \nabla_4 = \nabla^2 S$ , the trend and seasonal components can be assumed to be the outcomes of the models  $\nabla^2 x_{mt} = \theta_m(B) a_{mt}$ , and  $S x_{st} = \theta_s(B) a_{st}$ , respectively; the irregular component can be assumed to be white noise,  $x_{ut} = a_{ut}$ . Thus, letting  $\theta(B) = (1 - \theta_1 B)(1 - \theta_4 B^4)$ , consistency with the overall model (i.e., equation (3.7)) implies

$$\theta(B) a_t = S \theta_m(B) a_{mt} + \nabla^2 \theta_s(B) a_{st} + \nabla \nabla_4 a_{ut}. \quad (5.1)$$

Since the l.h.s. of (5.1) is an MA of order 5, we can set  $\theta_m(B)$  and  $\theta_s(B)$  to be of order 2 and 3, respectively, so that the three terms in the r.h.s. of (5.1) are also of order 5. The component models will then be of the type:

$$\nabla^2 x_{mt} = (1 + \theta_{m,1} B + \theta_{m,2} B^2) a_{mt}, \quad (5.2.a)$$

$$S x_{st} = (1 + \theta_{s,1} B + \theta_{s,2} B^2 + \theta_{s,3} B^3) a_{st}, \quad (5.2.b)$$

and  $x_{ut} = a_{ut}$ . Equating the covariances of the l.h.s. and the r.h.s. of (5.1), a system of 6 equations is obtained. These equations express the relationship between the parameters of the overall model and the unknown parameters in the component models. Since the number of the latter is 8 ( $\theta_{m,1}$ ,  $\theta_{m,2}$ ,  $\theta_{s,1}$ ,  $\theta_{s,2}$ ,  $\theta_{s,3}$ ,  $V_m$ ,  $V_s$ ,  $V_u$ ), there is an infinite number of structures of the type (5.2) that are compatible with the same model (2.1). The identification problem is similar to the one that appears in standard econometric models (see, for example, Fisher, 1966). The model for the observed series is the reduced form, whereas the models for the components represent the associated structural form. For a particular reduced form, there is an infinite number of structures from which it can be generated. In order to select one, additional information has to be incorporated.



The traditional approach in econometrics has been to set a priori some parameters in the structural model equal to zero. Identification by zero-coefficient restrictions in unobserved component models has been analyzed in Hotta (1989). The  $i$ th component model is identified if the order of its AR polynomial ( $p_i$ ) exceeds that of its MA polynomial ( $q_i$ ). In fact, setting, for example,  $\theta_{m,2} = 0$  and  $\theta_{s,3} = 0$ , the system of 6 equations has now 6 unknowns, and the model becomes identified. This way of reaching identification is the approach most widely used in practice. For example, the STS decomposition sets a priori components with  $p_m = 2 > q_m = 1$  and  $p_s = 3 > q_s = 1$ ; Pauly (1989) sets  $p_i = q_i + 1$  for all components (apart from the noise  $x_{ut}$ ); in econometrics, the practice of using random-walk trends (with  $p = 1 > q = 0$ ) guarantees that the trend component is identified. But if traditional econometric models rationalize setting a priori some coefficients equal to zero on the grounds of economic theory (for example, some variable may not affect demand), no such rationalization holds for the unobserved component case. However, despite the lack of a priori information on the components MA parameters, other types of considerations may be brought into the picture:

Watson (1987), for example, proposed a conservative solution (see also Findley, 1985): Consider all possible model specifications (under Assumptions 1-5) that satisfy the identity (5.1), and have nonnegative component spectra. For a given model for the observed series, they form the set of "admissible" decompositions. Different admissible decompositions have different model specifications for the components, and the mean square error (MSE) of the component estimators (as we shall see later) will also differ. Watson proposes a "minimax" strategy, whereby the Minimum MSE estimator of the component is obtained for the admissible decomposition with maximum estimation error. As another example, the AMB approach uses the following consideration:

In the decomposition of model (2.1) into three orthogonal components as in (5.2), the sum of the components spectra should be equal to the spectrum for the observed series. Figure 9 displays (with the continuous line) the spectra of the components of an admissible decomposition for a particular case of model (2.1), namely that given by (6.1). Let  $g_m(\omega)$  denote the trend component spectrum. By noticing that its minimum is  $g_m(\pi) = .1$ , it follows that  $x_{mt}$  can be further decomposed into a trend and an orthogonal white-noise irregular, with the variance of the latter in the interval  $(0, .1)$ . Removing white noise with variance .07 from  $x_{mt}$ , and assigning this noise to the irregular component, another admissible decomposition is obtained, and it is given by the dotted line in figure 9 (the seasonal component has not changed). If  $x_{mt}^1$  and  $x_{mt}^2$  denote the trends in the first and the second decompositions, respectively, then  $x_{mt}^1$  can be expressed as  $x_{mt}^1 = x_{mt}^2 + n_t$ , where  $n_t$  is white noise, orthogonal to  $x_{mt}^2$ . Thus the trend  $x_{mt}^1$  can be seen as obtained from the trend  $x_{mt}^2$  by simply adding white noise. The latter trend would seem preferable since it contains less noise. The previous consideration leads to the idea of choosing, within the set of admissible decompositions, the one that provides the smoothest trend.

Since the spectrum of a trend component should be monotonically decreasing in  $\omega$ , its minimum will be obtained for  $\omega = \pi$ . If this minimum is larger than zero, further

white noise could still be removed from the trend. Therefore, the noise-free condition implies  $g_m(\pi) = 0$ , which is equivalent, in the time domain, to the presence of the root  $B = -1$  in the polynomial  $\theta_m(B)$ . Components from which no additive noise can be extracted were first proposed by Box, Hillmer, and Tiao (1978), and Pierce (1978); they have been termed "canonical components". The canonical component has the important property that any other admissible component can be seen as the canonical one plus superimposed (orthogonal) noise. If an admissible decomposition exists, moreover, the canonical requirement identifies the component, since the canonical component is uniquely obtained by simply subtracting from any admissible component spectrum its minimum. As Hillmer and Tiao (1982) show, the canonical condition also minimizes the variance of the component  $p$ -innovation  $a_{mt}$ ; since  $a_{mt}$  is the source of the stochastic variability, the canonical component can be seen as the closest to a deterministic component that is compatible with the stochastic structure of the series. (Some additional interesting properties of canonical components will be seen in section 10.)

It is worth noticing that the random-walk trend, popular in econometrics, is not a canonical component. In particular, for the model  $\nabla x_t = a_t$  with  $V_a = 1$ ,  $x_t$  can be decomposed as in (1.2), with  $m_t$  and  $n_t$  orthogonal, the first given by

$$\nabla m_t = (1 + B) a_{mt} \quad (V_m = .25), \quad (5.3)$$

and  $n_t$  white noise, with variance  $V_n = .25$ . Model (5.3) is the canonical trend within a random-walk trend.

Back to the admissible decomposition of figure 9, and noticing that the spectrum of the seasonal component has a positive minimum, it follows that that component can, in turn, be expressed as the sum of a smoother seasonal component and an orthogonal white noise. Again, maximizing the smoothness of the component leads to a noninvertible component model. As with the trend, any admissible seasonal component can be seen as the canonical one with superimposed noise, and hence, if an admissible component is available, the canonical one can be trivially obtained. Finally, if, in the decomposition of (2.1), the canonical trend and seasonal components are specified, then the variance of the irregular (white-noise) component is maximized.

In the time domain analysis, the canonical requirement replaces the zero-coefficient restrictions with more general constraints among the coefficients. A canonical trend implies  $\theta_m(B = -1) = 1 - \theta_{m,1} + \theta_{m,2} = 0$ , and if, for example, the spectral zero of the canonical seasonal component occurs at  $\omega = 0$ , then  $\theta_s(B = 1) = 1 + \theta_{s,1} + \theta_{s,2} + \theta_{s,3} = 0$ . The two constraints, added to the system of 6 covariance equations associated with (5.1), provide now a system of 8 equations which can be solved for the 8 unknown parameters, and hence the model becomes identified. Although I shall often use canonical decompositions for illustration, the general discussion that follows is valid for any unobserved components model under Assumptions 1-5, independently of the particular identification criteria.



## 6 An Example

### a) The Quarterly US GNP Series

I shall illustrate some of the previous discussion and, in particular, the AMB decomposition with the series that has attracted more attention in the business cycle (and permanent component) literature: the quarterly series ( $y_t$ ) of US GNP. I shall consider the original not seasonally adjusted, series, and seasonality will explicitly be a part of the model. (The series contains 140 observations, from Jan 51 to Dec 85, and was kindly supplied to me by Fabio Canova.) Letting  $x_t = \log y_t$ , the model

$$\nabla \nabla_4 x_t = (1 - .702 B^4) a_t, \quad (6.1)$$

provides a very good fit. The residual standard deviation is  $\sigma_a = .015$ , and the only anomaly is a slightly high Kurtosis value of 4.02 ( $SE = .42$ ), due to two outlier observations, of opposite signs, for 1958/1 and 1984/1. Following the procedure of Chen and Liu (1993), the two are identified as temporary changes. Correcting for the outliers, the model varies very little with respect to (6.1). Since the two outliers are of moderate size and have little effect on the model, for illustration purposes we opt for the uncorrected series.

Model (6.1) is a particular case of (2.1), and hence the series  $x_t$  can be expressed as the sum of a trend, a seasonal, and an irregular (mutually uncorrelated) components, with models as in (5.2). We proceed to show a simple way to obtain, from (6.1), the unknown parameter values of (5.2). Let,  $\zeta(B)$  denote a finite polynomial in  $B$ , and denote by  $\gamma(\zeta, B)$  the product

$$\gamma(\zeta, B) = \zeta(B) \zeta(F) = \gamma_0 + \sum_j \gamma_j (B^j + F^j).$$

( $\gamma$  is the AutoCovariance Generating Function, ACGF, of the process  $\zeta(B) e_t$ , with  $V_e = 1$ ). Denote by  $G(\zeta, \omega)$  the Fourier transform of  $\gamma$ , that is

$$G(\zeta, \omega) = g_0 + \sum_j g_j \cos(j\omega),$$

where  $g_0 = \gamma_0$ , and  $g_j = 2\gamma_j$  ( $j \neq 0$ ). (Note that  $G$  is the spectrum of the same process  $\zeta(B) e_t$ .) From (5.2) and (3.5) we have the identity:

$$\frac{G(\theta, \omega)}{G(\phi, \omega)} = \frac{G(\theta_m, \omega)}{G(\nabla^2, \omega)} + \frac{G(\theta_s, \omega)}{G(S, \omega)} + k, \quad (6.2)$$

where the only unknowns are the parameters in

$$\begin{aligned} G(\theta_m, \omega) &= g_{m,0} + g_{m,1} \cos \omega + g_{m,2} \cos 2\omega \\ G(\theta_s, \omega) &= g_{s,0} + g_{s,1} \cos \omega + g_{s,2} \cos 2\omega + g_{s,3} \cos 3\omega, \end{aligned}$$

and the constant  $k = V_u$ . Removing denominators in (6.2), and using the relationship  $\cos(r_1 \omega) \cos(r_2 \omega) = \{\cos(r_1 - r_2)\omega + \cos(r_1 + r_2)\omega\}/2$ ,  $r_1 > r_2$ , an identity is obtained

between two harmonic functions of the type  $\sum_{j=0}^5 g_j \cos(j\omega)$ . The function in the l.h.s. of the identity is known, and the one in the r.h.s. contains the unknown parameters. Equating the coefficients of  $\cos(j\omega)$ ,  $j = 0, \dots, 5$ , in both sides of the identity yields a linear system of 6 equations in 8 unknown parameters. A simple way to obtain a first solution is by setting, for example,  $\theta_{m,2} = \theta_{s,3} = 0$ , which implies  $g_{m,2} = g_{s,3} = 0$ . Solving now the system of equations yields

$$\begin{aligned} G(\theta_m^0, \omega) &= 1.424 - 1.418 \cos \omega, \\ G(\theta_s^0, \omega) &= .036 + .044 \cos \omega + .014 \cos 2\omega \end{aligned} \quad (6.3)$$

and  $k = V_u^0 \doteq 0$ . Replacing these values in (6.2), a first decomposition is obtained; since none of the three spectra in the r.h.s. of (6.2) is negative for  $\omega \in [0, \pi]$ , the decomposition is admissible. It is given by

$$\begin{aligned} g_m^0(\omega) &= G(\theta_m^0, \omega)/G(\nabla^2, \omega) \\ g_s^0(\omega) &= G(\theta_s^0, \omega)/G(S, \omega) \end{aligned} \quad (6.4)$$

and  $V_u^0 \doteq 0$ . Let  $k_m = \min g_m^0(\omega)$  and  $k_s = \min g_s^0(\omega)$ , for  $\omega \in [0, \pi]$ . Then the canonical trend and seasonal components are obtained through

$$\begin{aligned} g_m(\omega) &= g_m^0(\omega) - k_m \\ g_s(\omega) &= g_s^0(\omega) - k_s \end{aligned}$$

and  $V_u = V_u^0 + k_m + k_s$ . To find the ARIMA expression for each component, one simply needs to factorize the corresponding spectrum (an easy and accurate procedure for spectral factorization is described in Appendix A of Maravall and Mathis, 1993). It should be pointed out that knowledge of the component models, although of interest, is not needed in order to obtain the component estimation filter, which can be trivially obtained from the spectra (see section 7).

For the US GNP series, the models obtained for the canonical components are:

$$\nabla^2 x_{mt} = (1 + .085B - .915B^2) a_{mt}, \quad V_m = .194V_a \quad (6.5.a)$$

$$S x_{st} = (1 + .996B + .338B^2 - .456B^3) a_{st}, \quad V_s = .009V_a \quad (6.5.b)$$

and  $V_u = .182V_a$ . The trend MA polynomial can be factorized as  $(1 - .915B)(1 + B)$ , and hence the trend spectrum has a minimum of zero for  $\omega = \pi$ . The zero in the seasonal component spectrum occurs for  $\omega = .76\pi$ . As the variances of the  $p$ -innovations indicate, the series is characterized by a relatively strong stochastic trend, and a fairly stable seasonal component. Since the variance of the white-noise irregular is 18% of the variance of the one-period-ahead forecast error of the series, the stochastic variability of the seasonal and (in particular) of the trend component contribute in an important way to the error in forecasting the series.

The model for the seasonally adjusted series,  $x_{dt}$ , is easily derived from  $x_{dt} = x_{mt} + u_t$ . It is an IMA(2, 2) model, given by

$$\nabla^2 x_{dt} = (1 - .921B + .005B^2) a_{dt}, \quad V_d = .783V_a. \quad (6.5.c)$$



b) Some Comments on the Specification of the Component Model

1. From (6.3) and (6.4), the first admissible decomposition yielded the trend spectrum

$$g_m^0(\omega) = \frac{1.424 - 1.418 \cos \omega}{6 - 8 \cos \omega + 2 \cos 2\omega},$$

which, upon factorization, implies the IMA(2, 1) model:

$$\nabla^2 x_{mt}^0 = (1 - .912B) a_{mt}^0, \quad V_m^0 = .777 V_a.$$

The model is as (4.6), and can be expressed as an STS second-order random walk model of the type (4.5), with  $V_m = .709V_a$  and  $V_\mu = .006V_a$ . Since estimation may well indicate that  $V_\mu$  can be accepted as zero, the trend model would again be given by the random-walk plus drift model, in accordance with the results in Harvey and Jaeger (1991). Notice, however, that restricting the order of the MA implies a considerable increase in the variance of the  $p$ -innovation when compared to that for the canonical trend (.777 versus .194); the canonical trend is therefore considerably smoother.

2. Compatibility with the observed series model implies that an overall ARIMA model with  $\nabla \nabla_4$  in its AR part will always produce an I(2) trend. Yet if we consider (6.5.a), factorize the MA part, and cancel the MA root .915 with one of the unit AR roots, the trend model becomes  $\nabla x_{mt} = (1 + B) a_{mt} + \mu$ , an I(1) model, similar to the "trend in a random-walk trend" of equation (5.3). Furthermore, looking at the model for the seasonally adjusted series given by (6.5.c), it is immediately seen that it can be approximated by  $\nabla x_{dt} = a_{dt} + \mu$ , and hence is very close to the standard "random-walk plus drift" trend used in econometrics.

The near cancellation of a unit root in the trend of ARIMA models with a  $\nabla \nabla_s$  stationarity-inducing transformation, is often found in practice, and explains the apparent discrepancy between the I(1) models of econometricians and the I(2) trends of most statistical decompositions (see Maravall, 1993a). For the usual number of observations in quarterly or monthly series, it is most unlikely that sample information can reliably discriminate between the two models:

$$\nabla^2 m_t = (1 - .92B) a_{mt} \quad (6.6)$$

$$\nabla m_t = a_{mt} + \mu. \quad (6.7)$$

Model (6.6) can be expressed as model (6.7) by replacing the constant slope  $\mu$  by a slowly changing  $\mu_t$ . This flexibility is achieved at the cost of losing one observation, due to the additional differencing. Be that as it may, the short-term adaptability of the slope makes model (6.6) more suitable for short-term analysis; on the other hand, it is likely that this short-term flexibility is unsuitable for long-term inference. Since statistical practical applications are aimed at short-term monitoring, while the applications of unobserved component models by econometricians look at longer-term horizons, the use of specification (6.6) by the former and (6.7) by the latter seems justified. Still, the use of ARIMA models for long-term inference in economics has often been questioned; see, for

example, Cochrane (1988), Quah (1990), Christiano and Eichenbaum (1989), Diebold and Rudebush (1991), and Maravall (1993b).

3. Setting, without loss of generality,  $V_a = 1$ , it follows that, in the canonical decomposition of model (2.1), although the model for the trend depends on two parameters, the model for the seasonal component on 3, and the model for the irregular on 1, all those parameters are simply functions of  $\theta_1$  and  $\theta_4$ . It can be seen that different values of  $\theta_1$  and  $\theta_4$  have little effect of the MA parameters of the trend and seasonal component models, and a strong effect on the variance of the component  $p$ -innovations. More stable trends (i.e., larger values of  $\theta_1$ ) yield smaller values of  $V_m$ , and more stable seasonal components (i.e., larger values of  $\theta_4$ ) yield smaller values of  $V_s$ .

4. The AMB decomposition of the GNP series is meant to represent a reasonable decomposition; other specifications may be reasonable as well. What seems clear, however, is that the series does not contain much evidence of periodic cycles; there is none in the model, none in the residuals.

## 7 Optimal Estimation of Unobserved Components

### a) Minimum Mean-Squared Error Estimators

For the model consisting of equation (1.1) and the set of assumptions 1–5, the next assumption defines the estimator of interest.

**Assumption 6:** Denote by  $X_T = [x_1, \dots, x_T]$  the series of available observations. The optimal estimator of the unobserved component  $x_{it}$  is given by

$$x_{it|T} = E(x_{it}/X_T). \blacksquare$$

Assumption 6 is a standard assumption in model-based estimation of unobserved components; together with the other assumptions, it implies that  $x_{it|T}$  is a linear projection and will be the MMSE estimator.

There are two well-known procedures to compute the above conditional expectation. One is based on the Kalman filter; the other, on the Wiener–Kolmogorov (WK) filter. Both were first derived for stationary series (see, for example, Whittle, 1963, and Anderson and Moore, 1979), and subsequently extended to the nonstationary case (see Cleveland and Tiao, 1976; Bell, 1984; Ansley and Kohn, 1985; Maravall, 1988a; De Jong, 1988; among others).

The Kalman filter approach starts by setting the model in a state-space format, and runs a set of recursions after having established appropriate starting conditions. (For nonstationary models, those conditions have been the subject of considerable research; see Kohn and Ansley, 1986; De Jong, 1991; Bell and Hillmer, 1991; and Gómez and Maravall, 1993.) The Kalman filter provides an easy to program, computationally efficient algorithm, and is used in estimation of unobserved components in, for example, the approaches of Harrison and Stevens (1976), Engle (1978), Gersch and Kitagawa



(1983), Burridge and Wallis (1985), Dagum and Quenneville (1993) and, in general, in the STS methodology. It is also the standard procedure in most econometric applications; a good general reference is Harvey (1989). Although less popular, the WK filter is also used on occasion. Examples are found in Nerlove, Grether and Carvalho (1979), Sargent (1987) and, in particular, in the AMB methodology (see Maravall and Pierce, 1987). Ultimately, the two filters provide computationally efficient ways to obtain the same linear projection (that implied by the conditional expectation of Assumption 6). The WK filter offers the advantage of providing more information on the structure and functioning of the filter and is better suited for analytical discussion; it shall be used for the rest of the paper. The discussion, however, will also apply to components estimated with the Kalman filter, although there may be some small discrepancies due to the effect of different starting conditions.

## b) The Wiener-Kolmogorov Filter

When considering estimation of a component, it will prove convenient to work with the two component representations (1.2), where  $m_t$  denotes the component of interest, and  $n_t$  is the sum of the remaining components. The two components follow the models:

$$\phi_m(B) m_t = \theta_m(B) a_{mt}, \quad (7.1.a)$$

$$\phi_n(B) n_t = \theta_n(B) a_{nt}, \quad (7.1.b)$$

and (3.6) and (3.7) become  $\phi(B) = \phi_m(B) \phi_n(B)$ , and  $\theta(B) a_t = \theta_m(B) \phi_n(B) a_{mt} + \theta_n(B) \phi_m(B) a_{nt}$ . It will facilitate the presentation to begin by considering the case of a complete realization of the series  $x_t$ , extending from  $t = -\infty$  to  $t = \infty$ . Denote this realization by  $X$ . Assume, first, the case of a stationary series (and hence stationary components), and write (3.5) and (7.1) as  $x_t = \Psi(B) a_t$ ,  $m_t = \Psi_m(B) a_{mt}$ ,  $n_t = \Psi_n(B) a_{nt}$ . Then, the WK filter is given by

$$\hat{m}_t = m_{t|\infty} = E(m_t|X) = k_m \frac{\Psi_m(B) \Psi_m(F)}{\Psi(B) \Psi(F)} x_t, \quad (7.2)$$

where  $k_m = V_m/V_a$ . Replacing the  $\Psi$ -polynomials by their rational expressions, after cancellation of roots, it is obtained that

$$\hat{m}_t = \nu(B, F) x_t, \quad (7.3.a)$$

$$\nu(B, F) = k_m \frac{\theta_m(B) \theta_m(F) \phi_n(B) \phi_n(F)}{\theta(B) \theta(F)}, \quad (7.3.b)$$

where  $\nu(B, F)$  is the WK filter. It is seen that no AR roots appear in the denominator of the filter, which, under Assumption 3, will always converge. In fact, expression (7.3) also yields the optimal estimator of  $m_t$  in the (unit roots) nonstationary case. Direct inspection of  $\nu(B, F)$  shows that the filter is centered at  $t$ , symmetric, and convergent in  $B$  and  $F$ . In particular, the filter will be finite when the overall model (3.5) is a finite AR process. Expression (7.3.b) shows that the filter is precisely the ACGF of the ARIMA model

$$\theta(B) z_t = \theta_m(B) \phi_n(B) b_t, \quad (7.4)$$

where  $\text{Var}(b_t) = k_m$ . Assumption 3 guarantees stationarity; as for invertibility, the filter that yields  $m_t$  will be noninvertible when  $n_t$  is nonstationary.

Since  $\nu(B, F)$  is a 2-sided filter, it will be subject to the problem of preliminary estimation and revisions mentioned in section 1. This problem will be addressed in section 9; preliminary estimation typically affects a few years at the beginning and at the end of the series, and the historical estimator can be assumed to apply to the center years. In the quarterly US GNP example, 95% of the variance of the revision in the trend concurrent estimator has been completed after 3 years of data. Thus, in the 35 years of available data, the historical estimator can be assumed to be approximately valid for the 116 central observations.

By construction, the WK filter adapts itself to the series under consideration, and this adaptability avoids the dangers of under and overestimation mentioned in section 2, and associated with ad-hoc filtering. As an illustration, if the AMB method is used in the two extreme cases of unstable seasonality and unstable trend of figure 6, then figure 7 becomes figure 10. For the series with a highly stochastic seasonal, the filter adapts to the width of the seasonal peak, and the seasonally adjusted series does not display any spurious spectral peaks, as was the case in figure 7b. For the unstable trend case, figure 8 also displays the AMB trend filter. From its closeness to the spectral peak around  $\omega = 0$  in the series model, it is apparent that no spurious cycle will be induced in the detrended series.

It is worth mentioning that many ad-hoc filters, including the HP and the X11 ones, have been given an (approximate) model-based interpretation under Assumptions 1–6. Examples can be found in Cleveland and Tiao (1976), Tiao (1983), Burridge and Wallis (1984), King and Rebelo (1993), and Cogley (1990). For a symmetric ad-hoc filter, it will be in general possible to find an approximation derived from a model-based approach under Assumptions 1–6.

## 8 The Structure of the Optimal Estimator

### a) The Model for the Estimator

Consider the optimal estimator (7.3), with  $\nu(B, F)$  given by (7.4). Using (3.5), the estimator of  $m_t$  can be expressed in terms of the innovations ( $a_t$ ) in the observed series as

$$\phi_m(B) \hat{m}_t = \theta_m(B) \alpha_m(F) a_t, \quad (8.1.a)$$

where  $\alpha_m(F)$  is the (invertible) forward filter

$$\alpha_m(F) = k_m \frac{\theta_m(F) \phi_n(F)}{\theta(F)}. \quad (8.1.b)$$

Comparing (7.1.a) with (8.1.a), it is seen that the expressions for the unobserved component ( $m_t$ ), and for its estimator ( $\hat{m}_t$ ), share the same AR polynomial. The



stationarity-inducing transformation is the same for both, and component and estimator have the same order of integration. Moreover, the two models (7.1.a) and (8.1.a) share the same polynomials in the operator  $B$ . The basic difference between the two models is the presence of the polynomial  $\alpha_m(F)$  in the model for the estimator. This forward filter expresses the two-sided character of the WK filter, that is, the dependence of the final estimator  $\hat{m}_t$  on innovations posterior to period  $t$  (this dependence goes to zero as the time distance increases.)

In any event, the models for  $m_t$  and  $\hat{m}_t$  are structurally different. They will display different variances and covariances (for the stationary transformation), and different spectra. These differences are illustrated in figure 11, which compares the component and estimator spectra for the trend and seasonal components of the US GNP series. It is seen that the spectrum of a component is similar to that of its estimator, except for the dips displayed by the latter at the frequencies for which the other components present spectral peaks.

To understand the differences between the component spectrum and that of its estimator, from (7.3), the spectrum of  $\hat{m}_t$  is equal to

$$g_{\hat{m}}(\omega) = R^2(\omega) g_x(\omega), \quad (8.2)$$

where

$$R(\omega) = \frac{g_m(\omega)}{g_x(\omega)} = \frac{1}{1 + 1/r(\omega)}, \quad (8.3)$$

and  $r(\omega) = g_m(\omega)/g_n(\omega)$ . Since  $m_t$  is the component of interest, we shall refer to it as the signal; accordingly,  $n_t$  will be denoted the noise. Therefore,  $r(\omega)$  represents the signal-to-noise ratio, and MMSE estimation proceeds as follows: For each  $\omega$ , it computes the signal-to-noise ratio. If the ratio is high, then the contribution of that frequency in the estimation of the signal will also be high. Thus, for the US GNP example, if the trend is the signal, then  $R(0) = 1$ , and the frequency  $\omega = 0$  will only be used for trend estimation. Since the noise, in this case, contains seasonal nonstationarity, for the seasonal frequencies,  $R(\omega) = 0$ , so that these frequencies are ignored in computing the trend. The associated spectral zeroes in  $g_{\hat{m}}(\omega)$  explain the dips in the spectra of figure 11a; they also imply that model (8.1.a) is noninvertible. This noninvertibility of the estimator is also evident from the unit seasonal roots of  $\phi_n(B)$  in (8.1.b), which appear in the MA part of model (8.1.a).

#### b) Structural Underestimation and Bias Towards Stability

Since  $r(\omega) \geq 0$ , then  $0 \leq R(\omega) \leq 1$ , and considering that (8.2) and (8.3) imply  $g_{\hat{m}}(\omega) = R(\omega) g_m(\omega)$ , it follows that the estimator will always underestimate the component. The amount of that underestimation depends on the particular model under consideration. From (8.2) and (8.3), it can be seen that  $g_{\hat{m}}(\omega)/g_m(\omega)$  is an increasing function of  $V_m/V_a$ . Therefore, the relative underestimation will be large (i.e.,  $g_{\hat{m}}(\omega)/g_m(\omega)$  will be small) when the variance of the component innovation  $V_m$  is relatively small. As an illustration, table 1 presents, for the two examples of figure 6, the effects of underestimation of the seasonally adjusted series ( $m_t$ ) and of the seasonal component ( $n_t$ ), where

underestimation is measured as the ratio of the variance of the stationary transformation of the estimator to that of the component.

Table 1: Underestimation of the Component ( $V_a = 1$ )

	Stable $m_t$ , unstable $n_t$		Unstable $m_t$ , stable $n_t$	
	$m_t$	$n_t$	$m_t$	$n_t$
Variance of the component innovation	.08	.20	.78	.01
Estimator var. as a fraction of component var. (station. transform.)	.09	.77	.85	.08

It is seen that underestimation of the component will be particularly intense when the stochastic variability of the component is already small. Thus the estimator will always be biased towards producing a series more stable than the component. It is also worth noticing that departures between estimator and component will be large when the component is of little importance, and viceversa.

In the model-based approach, the possibility of deriving the model that generates the component estimator can be a useful tool for diagnosis. In a particular application, the theoretical variance and AutoCorrelation Function (ACF) of the stationary transformation of the estimator can be easily obtained from (8.1), and compared to those of the estimate actually obtained. As seen in Maravall (1987), large departures between the theoretical and empirical values would indicate misspecification of the overall model and, as a consequence, of the estimation filters employed.

As an illustration, table 2 compares, for the US GNP series example, the theoretical and empirical variances of the stationary transformation of the estimated components for two filters: one is a “correct” filter, given by the AMB approach applied to model (6.1); the other one is the AMB filter of the stable trend–unstable seasonal example of figure 5 (i.e., an “incorrect filter”). The variances have been standardized by dividing them by the variance of  $\nabla \nabla_4 x_t$ , and the reported Standard Errors (SE) are asymptotic approximations (under the assumption that the underlying model is the one generating the filter). The table clearly indicates that, for the correct filter, the empirical variances are in close agreement with the theoretical ones; on the contrary, the two variances strongly disagree when the incorrect filter is employed.

c) Covariance Between the Estimators

Since the sum of the components is equal to the sum of their estimators, the underestimation of the component implies that, while the crosscovariances between different



Table 2: Variance of the Stationary Transformation of the Estimator

		Correct Filter	Incorrect Filter
Trend Estimator	Theoretical Value	.102	.003
	Estimate (SE)	.111 (.012)	.029 (.0004)
Seasonal Estimator	Theoretical Value	.001	.143
	Estimate (SE)	.001 (.0001)	.208 (.017)
Irregular Estimator	Theoretical Value	.052	.001
	Estimate (SE)	.040 (.006)	.002 (.0001)
Seasonally Adjusted Series Estimator	Theoretical Value	.825	.009
	Estimate (SE)	.645 (.100)	.039 (.001)

components are always zero, this will not be the case for the estimators. For the two component decompositions (1.2), let  $\Gamma(B, F)$  denote the Crosscovariance Generating Function (CCGF) between  $\hat{m}_t$  and  $\hat{n}_t$ ; that is,  $\Gamma(B, F) = \sum_{j=-\infty}^{\infty} \gamma_j B^j$ , where  $\gamma_j = E(\hat{m}_t \hat{n}_{t-j})$ . Then, from (8.1) and the equivalent expression for  $\hat{n}_t$ ,

$$\Gamma(B, F) = (k_m k_n) \frac{\theta_m(B) \alpha_m(B) \theta_n(F) \alpha_n(B)}{\phi_m(B) \phi_n(F)} V_a,$$

or, after simplification,

$$\Gamma(B, F) = \frac{\theta_m(B) \theta_n(B) \theta_m(F) \theta_n(F)}{\theta(B) \theta(F)} (V_m V_n / V_a). \quad (8.4)$$

Therefore, the CCGF between the two estimators is symmetric and convergent; in particular, it is equal to the ACGF of the model

$$\theta(B) z_t = \theta_m(B) \theta_n(B) g_t, \quad (8.5)$$

where  $g_t$  is white noise with variance  $(V_m V_n) / V_a$ . Even when the components are non-stationary, the crosscovariances between the estimators are finite.

The discrepancy between theoretically uncorrelated components and the existence of nonzero crosscovariances between their MMSE estimators in the model-based approach has been a cause of concern (see, for example, Nerlove, 1964; Granger, 1978; and Garcia-Ferrer and Del Hoyo, 1992). This concern, however, should be somewhat limited: for a complete realization of the series, the fact that the crossvariance is finite implies that, when at least one of the components is nonstationary (overwhelmingly the case of applied interest), the crosscorrelation between the estimators is also zero. Thus, model-based MMSE estimators of uncorrelated components are also uncorrelated. (For a finite — not too short — realization, the crosscorrelations will not be exactly zero, but will likely be very small.)

It is nevertheless interesting to notice that, although the estimators (in levels) will be uncorrelated, their stationary transformations will be correlated. This peculiar feature can be exploited at the diagnostics stage in a manner similar to that used for the ACF of the estimators. Corresponding to the particular model at hand, the theoretical crosscorrelations between the stationary transformation of the estimators can be easily derived from (8.4) or (8.5), and then compared to the ones obtained empirically. As an illustration, table 3 displays the theoretical value and the estimate obtained for the lag-0 crosscorrelation between the components for the US GNP series, using the correct and incorrect filters of table 2. Again, for the correct filter, the theoretical and empirical values are quite close. Since the incorrect filter is aimed at capturing a more stable trend than the one present in the GNP series, it provides an estimator that does not capture all the series trend variability, which contaminates then the seasonal and irregular estimates.

**Table 3:** Crosscorrelation Between the Stationary Transformations of the Estimators

		Correct Filter	Incorrect Filter
Between Trend and Seasonal Estimators	Theoretical Value	- .22	- .30
	Estimate	- .21	- .63
Between Trend and Irregular Estimators	Theoretical Value	- .01	- .36
	Estimate	- .09	- .71

#### d) The Component Pseudoinnovation

Estimation of the component  $p$ -innovation,  $a_{mt}$ , can be of some interest (an example is found in Harvey and Koopman, 1992), and the model-based approach can provide optimal estimators of the component  $p$ -innovations,  $a_{mt}$ . Taking conditional expectations in (7.1.a), it is obtained that  $\phi_m(B)\hat{m}_t = \theta_m(B)\hat{a}_{mt}$ , and considering (8.1), after simplification, the optimal estimator of the  $p$ -innovation can be expressed as

$$\hat{a}_{mt} = V_m \frac{\theta_m(F)\phi_n(F)}{\theta(F)} a_t,$$

a convergent forward filter of the innovations in the observed series. Therefore, the estimator of the  $p$ -innovation has a stochastic structure quite different from that of the white-noise  $p$ -innovation in the model. In fact, comparing the filter above with (7.4), it is seen that the ACGF of the standardized  $p$ -innovation estimator is precisely the WK filter.

#### e) Implications for Econometric Analysis

The properties of the optimal estimator have two relevant implications for applied econometric work. One has to do with the use of seasonally adjusted series (also of trends and



of detrended series) in univariate or vector AR models, and in some commonly used unobserved component models. The second implication concerns the practice of identifying cycles by detrending seasonally adjusted series.

From expression (8.1), relating the estimator to the innovations in the observed series, it is obtained that

$$\theta(F) \phi_m(B) \hat{m}_t = k_m \theta_m(B) \theta_m(F) \phi_n(F) a_t. \quad (8.6)$$

The MA part in (8.6) will be noninvertible when  $\theta_m(B)$  and/or  $\phi_n(B)$  contain one or more unit roots. As seen before,  $\theta_m(B)$  will contain a unit root when  $m_t$  is a canonical component. More relevantly,  $\phi_n(B)$  will contain unit roots whenever  $n_t$  is nonstationary. Thus, for example, if  $m_t$  denotes the seasonally adjusted series of the US GNP example of section 6, then, as implied by (6.5.b),  $\phi_n(B)$  is the polynomial  $S$ . The same is true when  $m_t$  denotes the trend component. Further, if  $m_t$  denotes the seasonal component, then  $\phi_n(B) = \nabla^2$ ; finally, if  $m_t$  denotes the seasonally adjusted and detrended series (i.e., the irregular), then  $\phi_n(B) = \nabla \nabla_4$ . Therefore, the estimator of the seasonally adjusted series, of the trend, of the seasonal component, and of the irregular component will all be noninvertible. More generally, since, as was argued in section 4, for series exhibiting seasonality, typically  $S$  is included in the AR polynomial of the seasonal component, seasonally adjusted series and trend estimators will typically be noninvertible.

Noninvertibility of the estimator of a component (when some of the other components are nonstationary) is a property of model-based estimation satisfying assumptions 1-6, and hence is valid for the AMB as well as the STS approach. It will also characterize estimators obtained with ad-hoc filters for which a model-based interpretation with nonstationary components can be given. For example, the filter  $C_i(B)$  in (1.3) that provides the detrended series of the HP filter can be written as (see Cogley, 1990)

$$C_{HP}(B) = \alpha_{HP}(B, F) (1 - B)^2 (1 - F^2),$$

where  $\alpha_{HP}(B, F)$  is symmetric and convergent in  $B$  and  $F$ , and hence for series that are  $I(d)$  with  $d < 4$ , the detrended series will be noninvertible. For X11, the filter that provides the seasonally adjusted series can be expressed as (see Cleveland, 1972)

$$C_{X11}(B) = \alpha_{X11}(B, F) S(B) S(F),$$

where, again,  $\alpha_{X11}(B, F)$  is symmetric and convergent in  $B$  and  $F$ , and hence the adjusted series will also be noninvertible. (Fuller, 1976, p. 417, shows how least-squares linear filters that remove seasonal variation from a stationary series also induce zeroes for the seasonal frequencies in the spectrum of the adjusted series.)

An immediate implication of the noninvertibility property is that the seasonally adjusted series will not have a convergent AR representation, and hence to fit finite AR models to seasonally adjusted series will not be appropriate. Furthermore, a vector autoregression (VAR) model should not be used to model a vector of time series some of which have been seasonally adjusted. It follows thus that unit root tests based on

AR representations (such as the Augmented Dickey Fuller test), or tests for cointegration based on VAR representations (such as Johansen tests) should not be applied to seasonally adjusted series, when the adjustment procedure has produced noninvertible series. Given the reluctance, often encountered in applied econometrics, to deal explicitly with seasonality, the practice of using adjusted series when fitting AR or VAR models is a common one (for a few important references, see Maravall, 1993a). The effect of using AR models on noninvertible series can be serious, and is the result of truncating a nonconvergent series; in particular, the AR or VAR parameter estimators will be inconsistent.

The error incurred when AR models are fit to noninvertible seasonally adjusted series can be, both, insidious and devastating, since it may easily pass undetected. These two important features are easily illustrated with the (X11 seasonally adjusted) US GNP series itself, a series which has often been the victim of this misspecification error. Following standard procedure, I consider the entire series, without truncation to remove preliminary estimators. Taking first differences of the log of the series, the ACF converges fast, and a low-order model seems appropriate. In fact, a simple "AR(1) + constant" model provides a reasonable fit, as evidenced by the relatively clean ACF of the residuals. This low-order AR specification is often found in applied econometrics work (see, for example, Campbell and Mankiw, 1987, or Evans, 1989). The choice of the parsimonious AR(1) specification seems to be confirmed by looking at what happens when the order of the AR polynomial is increased to 2, 3, or 4 lags. The additional AR coefficients are not significant, and the residual variance and Box-Ljung-Pierce Q-statistics for the residuals ACF remain roughly the same; this is shown in the first 2 columns of table 4. (The small negative value for lag 4 reflects the negative lag-4 autocorrelation often induced by seasonal adjustment.) Thus it is easy to conclude that the AR(1) approximation is reasonable. Yet it is not. If the order of the AR is further and further increased, additional significant coefficients keep showing up for large lags. Simultaneously, the residual variance and Q-statistics tend towards zero. This is evidenced in the last 2 columns of table 4; notice that in the AR(13) model, 10 of the 13 coefficients are significant. (This behavior could be expected since, for noninvertible series, the Partial ACF does not converge.) The example illustrates, thus, on the one hand, the potentially devastating effects of noninvertibility (or close to noninvertibility) on estimation of AR models, and its perverse nature since, unless one is on the lookout, it may well pass undetected.

The noninvertibility of the seasonally adjusted series also affects the specification of some unobserved component models commonly used in business cycle analysis. An example is model (2) in Stock and Watson (1988), where income is the sum (as in (1.2)) of a permanent and a transitory component (mutually orthogonal); the permanent component ( $m_t$ ) follows a random walk plus drift model, as in (4.4), and the transitory component ( $n_t$ ) is a stationary AR(2) model. The series has been seasonally adjusted with X11, and hence the homogenous series consisting of the central years is noninvertible. On the contrary, the two components  $m_t$  and  $n_t$  in the r.h.s. of (1.2) are invertible,



Table 4: AR Fits to US GNP Seasonally Adjusted (rate of change)

	AR(1)	AR(4)	AR(9)	AR(13)
$\mu (\times 10^2)$	1.28 (.18)	1.38 (.25)	.83 (.31)	.79 (.32)
$\phi_1$	<u>.303</u> (.081)	<u>.310</u> (.085)	<u>.445</u> (.083)	<u>.382</u> (.087)
$\phi_2$	—	.039 (.088)	-.045 (.086)	-.097 (.091)
$\phi_3$	—	.054 (.087)	.131 (.084)	<u>.183</u> (.089)
$\phi_4$	—	-.152 (.084)	-. <u>302</u> (.084)	-. <u>414</u> (.090)
$\phi_5$	—	—	.143 (.084)	<u>.222</u> (.092)
$\phi_6$	—	—	.013 (.081)	-.047 (.086)
$\phi_7$	—	—	<u>.213</u> (.081)	<u>.245</u> (.083)
$\phi_8$	—	—	-. <u>362</u> (.085)	-. <u>445</u> (.090)
$\phi_9$	—	—	<u>.319</u> (.083)	<u>.349</u> (.092)
$\phi_{10}$	—	—	—	.030 (.091)
$\phi_{11}$	—	—	—	<u>.226</u> (.090)
$\phi_{12}$	—	—	—	-. <u>230</u> (.090)
$\phi_{13}$	—	—	—	<u>.203</u> (.085)
$V_a (\times 10^4)$	1.49	1.44	1.12	.97
$Q_{20}$	28.6	28.2	19.4	9.8

Significant AR coefficients are underlined.

and hence their invertible sum cannot be equal to a noninvertible series. Besides the specification constraints it implies, the use of the seasonally adjusted series is not likely to reduce the dimension of the model. As (8.1) indicates, the filtered series, even when the underlying component has a very simple structure, will follow a rather complicated model, and may contain nonzero coefficients at relatively high-order lags.

As mentioned before, business cycle analysis often uses seasonally adjusted data, which is further detrended. The seasonally adjusted and detrended series is then analysed, perhaps by fitting an ARMA model where the AR part may contain a cycle. This way of proceeding can be extremely misleading, as the following example illustrates.

Under Assumptions 1–6, consider a quarterly series which is the sum of a trend, a seasonal, and an irregular component. The model for the trend is given by (4.5), that for the seasonal component by (4.11), and the irregular is white-noise. The trend model is similar to that of the model-based version of the HP filter (King and Rebelo, 1993), and the seasonal component model is as in the STS model of Harvey and Todd (1983). This unobserved components model is, in fact, the type of model used in Gersch and Kitagawa (1983). The innovation variances are set equal to  $V_m = V_u/1600$  (the standard value in the HP filter),  $V_s = 2V_u$ , and  $V_u = 1$ , and hence the series contains a very stable trend and a highly moving seasonal; by construction, it does not contain any periodic cycle, and the seasonally adjusted and detrended series is simply white-noise. The model

implies that the observed series  $x_t$  follows a model of the type

$$\nabla \nabla_4 x_t = \theta(B) a_t, \quad (8.7)$$

where  $\theta(B)$  is of order 5. If the series is seasonally adjusted and detrended by removing the MMSE estimators of the trend and of the seasonal component, then the residual obtained is the MMSE estimator of the white-noise irregular component  $x_{ut}$ . (For this example, by construction, the MMSE estimator  $\hat{x}_{ut}$  will not be too far from what would be obtained after seasonally adjusting with X11 and removing the trend with the HP filter.) For a white-noise irregular component ( $x_{ut}$ ) extracted from model (8.7), proceeding as in section 8a it is found that its MMSE estimator can be seen as generated by the model

$$\theta(F) \hat{x}_{ut} = k_u (1 - F) (1 - F^4) a_t. \quad (8.8)$$

Thus  $\hat{x}_{ut}$  will have the ACF of a stationary process (in particular, that of the inverse or dual model of (8.7)). For the example considered, figure 12 exhibits the spectrum of  $\hat{x}_{ut}$ . It is certainly far from being that of white-noise, and it is of interest to notice the large spectral peak for a frequency between 0 and the first seasonal frequency, which implies a relatively important cycle (with a period of approximately 5 years) that may easily show up in ARMA or even AR fits to  $\hat{x}_{ut}$ . The cyclical behavior detected in this way is entirely spurious, since it was not in the series. Notice that the spuriousness, in this case, is due to an incorrect interpretation of the filtered residuals, and not to the use of inappropriate filters to detrend and seasonally adjust the series.

The example illustrates a fairly general result. For models containing  $\nabla_4$  as part of the stationary transformation, the MMSE of a white-noise component will typically present a spectrum with two pronounced peaks towards the center of the frequency ranges  $(0, \pi/2)$  and  $(\pi/2, \pi)$ . This is a result of the spectral zeroes at  $\omega = 0, \pi/2$ , and  $\pi$ , induced by the presence of  $\nabla_4$  in the MA part of the model for  $\hat{x}_{ut}$ . The first peak, of course, will be associated with a cyclical frequency, and will produce a spurious cyclical-type behavior. As a consequence, the two-step procedure of fitting a model to a seasonally adjusted and detrended series is inappropriate for detecting periodic cycles. If a cycle is suspected, it should be estimated on the observed series, in a joint model with the rest of the components.

## 9 Preliminary Estimator and Forecast

### a) General Expression

In the previous section, attention was centered on the historical estimator, i.e. on the estimator obtained applying the WK filter (7.3) to a complete realization on the series. As before, let  $X$  and  $X_T$  denote the complete and finite realization of the time series, respectively, and let  $m_t$  be the component or signal of interest. To project  $m_t$  on the



finite realization  $X_T$ , since  $X_T \subset X$ , the optimal estimator can be expressed as

$$\begin{aligned} m_{t|T} &= E(m_t|X_T) = E(E(m_t|X)|X_T) = \\ &= E(\hat{m}_t|X_T), \end{aligned} \quad (9.1)$$

which implies that  $m_{t|T}$  can be expressed as

$$m_{t|T} = \nu(B, F) x_{t|T}, \quad (9.2)$$

where  $\nu(B, F)$  is the WK filter given by (7.3.b),  $B$  and  $F$  operate on  $t$ , and  $x_{t|T} = E(x_t|X_T)$ . Since  $x_{t|T}$  is the forecast of  $x_t$  done at time  $T$  (equal to  $x_t$  if  $T \geq t$ ), the estimator (9.2) can be seen as the WK filter applied to the available series extended at both ends with forecasts and backcasts (i.e., applied to the "extended series"). For a large enough (positive)  $T - t$ , (9.2) provides in practice the final or historical estimator of  $m_t$ , equivalent to (7.3). As  $t$  approaches  $T$ , (9.2) provides preliminary estimators of recent signals; for  $t > T$ , (9.2) yields the  $(t - T)$ -periods-ahead forecast of the signal. Forecasts can thus be seen as particular cases of preliminary estimation (and can be computed in a simple recursive way; see Burman, 1980).

Consider the model for the observed series, given by (3.5), and let  $p$  and  $q$  be the orders of the AR and MA polynomials. Unless  $q = 0$ , the filter in (9.2) will still contain an infinite number of weights. Since the filter is convergent, it could be safely truncated and approximated by a finite filter of the type (1.4). (The finite approximation can also be viewed as the WK filter for the AR approximation to the invertible model (3.5).) In practice, however, there is no need to truncate the filter: The exact filter (7.3.b) can be applied in an efficient and easy manner using an algorithm due to Wilson and Burman (detailed in Burman, 1980). The algorithm requires only  $[q + \max(q, p)]$  forecasts and backcasts of the observed series, the solution of two sets of  $(p + q)$  linear equations, and some simple recursions. Besides its computational efficiency, the problem of the starting conditions is simplified since it reduces to the conditions associated with the assumptions  $E_t a_{t+k} = 0$  and  $E_t a_{t-k}^b = 0$  (for  $k > 0$ ), where  $a_t$  and  $a_t^b$  are the forward and backward innovations in the series. Thus the assumptions required are the same ones as those underlying ARIMA forecasting (see Box and Jenkins, 1970; Bell, 1984; and Brockwell and Davies, 1987).

To simplify the discussion, I shall consider the finite approximation (1.4) applied to the series extended with forecasts and backcasts, and assume  $T > 2r + 1$ , so that the estimator of the component for the center periods of the series can be taken as the historical estimator. For  $1 < t < r$ , the estimator will make use of the backcasts and will yield "preliminary" estimators for the starting periods. When  $T - r < t < T$ , the estimator will use forecasts of the series and will thus yield preliminary estimators for recent periods. The two types of preliminary estimators are mirror images of each other; I shall focuss attention on the preliminary estimator of recent periods, the one of applied interest.

It is easily seen (Box and Jenkins, 1970) that the forecast of the series ( $x_{t|T}$ ,  $t > T$ ) can be expressed as

$$x_{t|T} = \Pi_{(t-T)}(B) x_T, \quad (9.3)$$

where the subindex  $(t - T)$  indicates the dependence of the filter on the forecast horizon. Combining (9.2) and (9.3), the preliminary estimator of the component can also be expressed as

$$m_{t|T} = \lambda_{(t-T)}(B) x_T, \quad (9.4)$$

which has a relevant implication for applied work. Let the component of interest be the seasonally adjusted series. If the observed series is  $X_T = [x_1, \dots, x_T]$ , and the adjusted series used is  $[m_{1|T}, \dots, m_{T|T}]$ , as is standard procedure, then the adjusted series is nonstationary in the sense that, as (9.4) shows, the underlying linear process generating the latter has time-varying coefficients.

If (9.2) is rewritten as

$$m_{t|T} = \nu^{(0)}(B) x_T + \sum_{j>0} \nu_{T-t+j} x_{T+j|T},$$

when a new observation,  $x_{T+1}$ , becomes available, the forecast  $x_{T+1|T}$  is replaced by the observation, and the forecasts,  $x_{T+j|T}$ ,  $j > 1$ , are updated to  $x_{T+j|T+1}$ . The new estimator obtained will be  $m_{t|T+1}$ , and the one-period revision in the estimator is

$$d_{t|T}(1) = m_{t|T+1} - m_{t|T} = \sum_{j>0} \nu_{T-t+j} (x_{T+j|T+1} - x_{T+j|T}) = \left( \sum_{j>0} \nu_{T-t+j} \Psi_{j-1} \right) a_{T+1}, \quad (9.5)$$

and hence the one-period revision is a constant fraction of the series innovations. More generally, as seen in Pierce (1980), the revision between two periods  $T$  and  $T + K$  is an  $MA(K - 1)$  process, which can be expressed in terms of the innovations  $a_{T+1}, \dots, a_{T+K}$ .

When  $T \rightarrow \infty$ , the estimator  $m_{t|T}$  becomes the historical estimator  $\hat{m}_t$ . It will prove useful to rewrite expression (8.1) as

$$\hat{m}_t = \eta^{(0)}(B) a_T + \eta^{(1)}(F) a_{T+1}, \quad (9.6)$$

where the first term in the r.h.s. includes the effect of the starting conditions and of the innovations up to and including  $a_T$ , and the second term contains the innovations posterior to  $T$ . From (8.1.b), the filter  $\eta^{(1)}(F)$  is convergent; its weights are easily obtained as the coefficients  $\eta_{T-t+j}(j = 1, 2, 3, \dots)$  of the polynomial  $\eta(B, F)$  obtained through the identity

$$\theta_m(B) \theta(F) \eta(B, F) = V_m \theta_m(B) \theta_m(F) \phi_n(F)$$

(see Maravall, 1993b). From (9.6) and (9.1), the preliminary estimator is then equal to

$$m_{t|T} = \eta^{(0)}(B) a_T, \quad (9.7)$$

and subtracting this expression from (9.6), the full revision that the estimator will undergo is found to be:

$$d_{t|T} = \eta^{(1)}(F) a_{T+1}, \quad (9.8)$$

and hence, due to the invertibility of  $\theta(B)$ , the full revision is a stationary process. Of course, given that  $\eta^{(1)}(B)$  is determined from the ARIMA model for the series, the properties of the revision in a preliminary estimator will be different for different models.



## b) Some Applications of Interest

Among the many preliminary estimators, of particular relevance in economic policy making and monitoring is the so-called concurrent estimator, obtained when  $t = T$ . The concurrent estimator yields the estimator of the component for the most recent period, and is obtained with the one-sided filter  $m_{it} = \eta^{(0)}(B) a_t$ . (It is worth mentioning that, as shown in Watson, 1986, the Beveridge-Nelson decomposition can be obtained as the optimal concurrent estimator in a model-based framework under assumptions 1-6.)

In general, revisions are implied by the use of a two-sided filter; figure 3 illustrated, for example, the revisions in the concurrent trend estimator for the HP filter. In practice, revisions in components such as the trend and seasonal are of considerable importance for many macroeconomic series. An example is provided by Maravall and Pierce (1983), in the context of the conduct of monetary policy by the Federal Reserve during the period of the seventies. We compared the concurrent estimator of the seasonally adjusted M1 monthly growth with its final estimator, and with the tolerance bands set every month by the Federal Open Market Committee. We obtained that the concurrent estimator gave a false signal (in the sense of indicating unacceptable growth when the final estimator eventually showed that growth was within the tolerance range, or viceversa) 40% of the time. We further argued that this proportion could be reduced with improved seasonal adjustment methods; still, the percentage of false signals could not go below 20%. Thus an important percentage of false signals could be attributed to the single effect of the revision error in the concurrent estimator of the seasonal component.

Besides interest in estimating components for recent periods, on occasion it is the component forecast that is of interest. An example in the area of economic policy is the role played by the forecasts of the monetary aggregate seasonal component, used by the monetary authority in short-term control of the money supply. Ad-hoc filtering, as was mentioned in section 2, does not provide a framework for optimal forecasting of the component, and X11, for example, yields 12 forecasts of the seasonal component obtained with the fixed extrapolation formula:

$$s_{T+j|T} = s_{T-12+j|T} + (s_{T-12+j|T} - s_{T-24+j|T})/2; \quad j = 1, \dots, 12. \quad (9.9)$$

As seen previously, the model-based approach provides optimal forecasts of the components, obtained as a trivial extension of preliminary estimation. In the area of applied econometrics an example of unobserved component forecasting is the impressive work of Stock and Watson (1989, 1991, 1993) on forecasting the business cycle using a model-based unobserved component model. Although an ingenious and attractive approach, care should be taken when interpreting the results precisely because of the unobserved component structure of the model. This is illustrated with the following example, consisting of the simplest nontrivial case of Stock and Watson's model,

$$\nabla y_t = \nabla c_t + n_t \quad (9.10.a)$$

$$(1 - \phi B) \nabla c_t = b_t, \quad (9.10.b)$$

where  $n_t$  and  $b_t$  are independent white-noise variables. The unobserved component  $c_t$  represents “the state of the economy” and it is modeled as an  $I(1)$  trend. (Notice that, since the r.h.s. of (9.10.a) is the sum of two invertible processes, and hence invertible, once again the model cannot be applied to noninvertible seasonally adjusted series.) A recession (expansion) is then defined as a sequence of  $\nabla c_t$  that are below (above) a certain threshold. The definition is made, thus, in terms of the “true” unobserved component. (Associating the business cycle with the behavior of a trend is an important departure from the traditional procedure of identifying the business cycle on the detrended series, as in Watson, 1986, or Clark, 1987, and from the periodic stochastic cycle component of Harvey, 1985.)

Stock and Watson attempt to capture in their model the official dating of recessions by the NBER Business Cycle Dating Committee (BCDC). So, assuming that (9.10) actually duplicates BCDC behavior, since the component is never observed, the BCDC is forced to operate with the best possible estimator, that is with the historical estimator (BCDC behavior reveals, in fact, a two-sided filter, typical of a historical estimator). Finally, in order to obtain recession forecasts, Stock and Watson consider the joint distribution of future sequences of  $\nabla c_t$  conditional on the available information. By letting  $x_t = \nabla y_t$  and  $m_t = \nabla c_t$ , the model is seen to be a particular case of the model-based procedure under Assumptions 1–6: the simple “AR(1) + noise” decomposition. From previous results, the differences in the distributions of the component, the preliminary and the final estimator will imply that the probability of positive sequences of the component will be structurally different from that of the historical estimator (see Maravall, 1993b). Thus, a systematic bias will show up when matching the forecasted probabilities with the series of historical estimators (in accordance with what Stock and Watson find).

## 10 Estimation Errors and Inference

### a) Historical Estimation Error and Revision Error

Point estimators or forecasts of the components are of limited interest unless some information is provided about their precision. This information is unavailable when ad-hoc filters are applied. The model-based approach, on the contrary, provides, as we have seen, the full distribution of the estimator (historical or preliminary), so that the properties of the estimation error can easily be obtained.

Consider the two-component decomposition (1.2), and let  $m_t$  be the component of interest, whose estimator is given by (9.2). The estimation error is

$$e_{qt} = m_t - m_{qt}, \quad (10.1)$$

which can be rewritten as

$$e_{qt} = d_t + d_{qt}, \quad (10.2)$$

where  $d_t = m_t - \hat{m}_t$ , and  $d_{qt} = \hat{m}_t - m_{qt}$ . Thus,  $d_t$  is the error in the historical estimator, and  $d_{qt}$  is the revision error contained in the estimator  $m_{qt}$ . As shown in Pierce (1979),



the two errors  $d_t$  and  $d_{t|T}$  are independent (under assumptions 1-6), and the historical estimation error can be seen as generated by the stationary ARIMA model

$$\theta(B) d_t = \theta_m(B) \theta_n(B) g_t, \quad (10.3)$$

with  $V_g = V_m V_n / V_a$ . The variance of  $d_t$  is therefore finite; both, variance and ACF, can be easily computed from (10.3). (The ACF of the error is of interest when computing approximate standard errors for the rates of growth of the component.) As noticed in Maravall and Planas (1993), model (10.3) is identical to model (8.5), and hence the CCGF between the estimators  $\hat{m}_t$  and  $\hat{n}_t$  is the same as the ACGF of the historical estimation error of  $m_t$  (and of  $n_t$ ). This result has an implication of interest: When searching for a criterion to select a unique decomposition among the set of admissible ones, one could think of selecting the specification for which the (lag-0) crosscovariance between the two estimators is minimized, given that the components are assumed orthogonal. On the other hand, one may select the decomposition for which the historical estimation error is minimum. What the previous result tells us is that both criteria lead to the selection of the same decomposition. Moreover, Maravall and Planas show that, in the selected decomposition, one of the two components is always a canonical one.

As for the revision error,  $d_{t|T}$ , from expression (9.8) its properties (in particular variance and ACF) can be easily derived, for any pair  $(t, T)$ . From the orthogonality of  $d_t$  and  $d_{t|T}$ , the variance and ACF of the total estimation error  $e_{t|T}$  are, then, straightforward to obtain. Notice that the fact that  $d_t$  and  $d_{t|T}$  have finite variance implies that the theoretical component,  $m_t$ , its historical estimator,  $\hat{m}_t$ , and its preliminary estimator or forecast,  $m_{t|T}$  are all pairwise cointegrated.

#### b) An Example: The U.K. Money Supply Series

To illustrate the use of the model-based approach in inference, I consider an example within the area of monetary policy, where the need for a measure of the estimator's uncertainty has been repeatedly pointed out (see, for example, Bach et al., 1976; Moore et al., 1981; and Hibbert Committee, 1988). The example is the monthly series of the monetary aggregate  $M_1$  in the U.K. from January 1983 to December 1991; seasonal adjustment of the U.K. monetary aggregate has been an issue of recent concern (see Bank of England, 1992).

Letting  $x_t$  denote the log of the series, the model

$$\nabla \nabla_{12} x_t = (1 - .738B^{12}) a_t, \quad (10.4)$$

with  $\sigma_a = .00674$ , fits the series very well; its structure is the monthly equivalent of that of model (6.1), obtained for the quarterly GNP series. As was the case then, the model reveals a relatively stochastic trend and a fairly stable seasonal component. Using the AMB approach, and standardizing units by setting  $V_a = 1$ , the models for the trend and seasonally adjusted series are, respectively, given by the IMA(2, 2) models:

$$\begin{aligned} \nabla^2 x_{mt} &= (1 - .975B)(1 + B) a_{mt} & (V_m = .191) \\ \nabla^2 x_{dt} &= (1 - .975B)(1 - .004B) a_{dt} & (V_d = .768). \end{aligned}$$

Both are indistinguishable from an “ $I(1) + \text{drift}$ ” model and, in particular, the seasonally adjusted series can be seen as a random walk with a slowly changing drift. The seasonal component model is of the type  $Sx_{st} = \theta_s(B)a_{st}$ , where  $S = 1 + B + \dots + B^{11}$ ,  $\theta_s(B)$  is of order 11, and  $V_s = .024$ . The irregular component  $x_{ut}$  is simply white-noise with  $V_u = .189$ .

For this decomposition, the variance of the different estimation errors for the concurrent and historical estimators of the trend and of the seasonally adjusted series are displayed in table 5. For both components, the variance of the historical estimation error is close to that of the revision error; both types of error are smaller for the seasonally adjusted series than for the trend. (Since estimation of the trend requires additional removal of the irregular, it is sensible that its estimation error be larger.) The variance of the concurrent estimator of the trend is slightly smaller than 1/3 of the variance of the one-period-ahead forecast error of the series  $x_t$ ; this fraction becomes 1/4 for the seasonally adjusted series estimator.

**Table 5:** Variance of Estimation Errors: Concurrent Estimator (in units of  $V_a$ )

Type of Error	Trend	Seasonally Adjusted Series
Final Estimation Error	.169	.110
Revision Error	.163	.114
Total Estimation Error	.332	.224

Besides the magnitude of the revision error in the concurrent estimator, it is also of interest to know the duration of the revision period, that is, how many periods it takes for a new observation to no longer significantly affect the estimate. From (9.8) it is easily found that, for the example considered, after one more year of additional data, the variance of the trend revision error has decreased by 70%; for the seasonally adjusted series this percentage is 50%. After three years of additional data, 85% of the trend revision error variance has been removed; 77% for the seasonally adjusted series. After 5 years, the percentages become 92% for the trend and 88% for the adjusted series. In 5 years, thus, both estimators have practically converged. Notice that, despite its larger revision error in the concurrent estimator, the trend estimator converges faster than that of the seasonally adjusted series. Recalling that the trend component was highly stochastic, while the seasonal component was fairly stable, the example illustrates a general result: highly stochastic components are characterized by large revision errors which converge relatively fast, while the removal of stable components implies smaller revision errors, which tend to converge slowly. Thus the revision lasts long when the removed component is of little importance in explaining the series variability. This “compensation” effect (i.e., large revisions converge fast) is easily understood from the following consideration. As mentioned in section 3, the stability of a component is



associated with roots in  $\theta(B)$ , the MA part of the model for  $x_t$ , that are close to the unit AR roots. Due to these large roots,  $\theta(B)^{-1}$  will converge slowly and hence, according to (7.3.b), the estimation filter of stable components will be long. Moreover, since  $\theta(F)$  is also present in the denominator of  $\eta(B, F)$ , expression (9.8) implies that the revision in the estimator will last long.

We have seen convergence of the concurrent estimator to the historical one, it is also of interest to look at how fast the estimation error increases when we consider the forecasts. For the log of the M1 series, figure 13 presents the standard errors for the 1-to-12-periods-ahead forecasts, for the trend, seasonally adjusted and original series. The (small but consistent) gain in precision from using the trend in short-term forecasting is apparent. All considered, comparing the seasonally adjusted series and the trend component, although the concurrent estimator of the latter has a larger error variance, it converges faster to the final estimator, and it provides more precise forecasts. These are important features to consider when deciding which of the two components provides a more adequate signal to measure the underlying evolution of the series.

Assume now that at the beginning of a new year (with the last observation that of December), an annual money growth target is set. As time passes, new observations become available and it is of interest to see what growth this new data implies, once it has been cleaned of seasonality. The most frequent operating procedure is to compute, at the beginning of the year, seasonal factors for the next months (through (9.9) when X11 is used), and adjust incoming monthly data with these factors. (At some institutions, such as the Federal Reserve Board and the Bank of England, seasonal adjustment is done with a higher than once-a-year frequency.) There is an obvious loss of precision in using a forecasted factor instead of the one obtained every month with concurrent estimation. Since concurrent adjustment is a costly procedure for data-producing agencies (the series have to be constantly modified), as stressed by the Bank of England (1992), it is important to quantify the loss in precision associated with intermittent adjustment. This quantification is easily done in the model-based approach by comparing the error in the seasonal component concurrent estimator with the average of the errors for the forecasted seasonal component. For the example considered and the once-a-year versus concurrent adjustment comparison, it is found that concurrent adjustment produces an average reduction of 11.2% in the root MSE of the seasonal component estimator. This relatively small gain is explained by the fact that the seasonal component is fairly stable and displays relatively small forecasting errors.

As new monthly observations become available, the forecast of the annual rate of growth will be updated. The standard error associated with these updated forecasts for a fixed horizon (that of the end of the year) are precisely those of the forecasts displayed in figure 13. At the beginning of the year, the forecasted rate of growth for the year has approximately the same error whether the forecast is computed with the series, with the trend, or with the seasonally adjusted series; this standard error equals 2.35 percent points (p.p.) of growth. For intra-year updating, after 4 months, for example, the standard error of the rate of growth is down to 1.83 p.p. if the trend is used, and to 1.91

p.p. for the forecast computed with the observed series. After 8 months, these standard errors become 1.24 p.p. and 1.35 p.p..

Although, as pointed out in Box, Pierce, and Newbold (1987), the updated forecast of the trend rate of growth provides possibly the most natural tool for monitoring the underlying evolution of the series throughout the year, standard operating procedures rely heavily on a battery of different rate-of-growth measurements. Using linear approximations, the model-based approach can be used to derive the variances of the associated estimation errors. Consider, for example, the concurrent estimator of the most commonly used rate: the monthly rate of growth of the monthly estimator (typically, of the seasonally adjusted series), and denote this rate by  $R$ . (The wording of the Federal Open Market Committee Record of Policy Action, for example, stated that modifications to the Federal funds rate had to be based on the evolution of this month-to-month rate of growth of the seasonally adjusted series.) If capital letters denote levels and small letters denote logs, the rate can be expressed as

$$R_t = (M_t - M_{t-1})/M_{t-1} \doteq m_t - m_{t-1},$$

where  $m_t$  is the signal of interest in the decomposition of (10.4). The concurrent estimator of  $R_t$  is

$$R_{t|t} = m_{t|t} - m_{t-1|t}.$$

Consider the auxiliary rate  $R_A = m_{t|t} - m_{t-1|t-1}$  and its estimation error,  $e_t^A$ , which can be expressed as:

$$e_t^A = R_t - R_A = (R_t - R_{t|t}) + (R_{t|t} - R_A). \quad (10.5)$$

The second parenthesis in the r.h.s. of (10.5) is equal to  $R_{t|t} - R_A = -(m_{t-1|t} - m_{t-1|t-1})$ , and represents, thus, the 1-period update in the concurrent estimator of  $m_{t-1}$ . From (9.5),  $R_{t|t} - R_A = r_{t-1|t}(1) = -\eta_1 a_t$ , where  $\eta_1$  is the coefficient of  $F$  in the polynomial  $\eta^{(1)}(F)$  of (9.8). For the M1 example,  $\eta_1 = .300$  when  $m_t$  is the trend, and  $\eta_1 = .112$  when  $m_t$  is the seasonally adjusted series. The first parenthesis in the r.h.s. of (10.5) is the estimation error in  $R_{t|t}$ ; denote this error by  $e_t^R$ . Since this error is the sum of the historical estimation error and a revision error which is a linear filter of innovations  $a_{t+k}$ ,  $k > 0$ , it follows that the two expressions in parenthesis in (10.5) are independent, and hence the variances will satisfy

$$V(e_t^R) = V(e_t^A) - \eta_1^2 V_a. \quad (10.6)$$

To obtain  $V(e_t^A)$ , notice that  $e_t^A$  can be expressed as  $e_t^A = (m_t - m_{t|t}) - (m_{t-1} - m_{t-1|t-1})$ , and hence is equal to the difference between two consecutive concurrent estimation errors  $e_{t|t}$  and  $e_{t-1|t-1}$  of (10.1). To simplify notation, let  $[e_t]$  denote the series of total estimation errors  $[e_{t|t}]$ . Then,

$$V(e_t^A) = V(e_t - e_{t-1}) = 2(1 - \rho_1^e) V_e, \quad (10.7)$$

where  $V_e$  is the variance of the total estimation error  $e_{t|t}$ , and  $\rho_1^e$  its lag-1 autocorrelation. As described earlier, both parameters can be easily obtained and for the M1 example, it is



found that  $V_e = .333 V_a$ ,  $\rho_1^e = .574$ , when  $m_t$  is the trend component, and  $V_e = .224 V_a$ ,  $\rho_1^e = .584$ , when  $m_t$  is the seasonally adjusted series. Inserting (10.7) in (10.6), the variance of the error in the concurrent estimator of  $R_t$  is  $V(e_t^R) = 2(1 - \rho_1^e) V_e - \eta_1^2 V_a$ .

In order to compare  $R_t$  with the annual target, the rate is multiplied by 12 and expressed in p.p. The standard errors of the rate computed in this way, for the trend and for the seasonally adjusted series, are given in the first row of table 6. For both components, the standard error is close to 3.5 p.p., and hence, at the 95% confidence level, a measurement of (say) 10% growth would be compatible with a target between, roughly, a 3% and a 17% annual growth. The implied range is certainly wider than the tolerance ranges used in practice. The rate  $R_t$  is therefore too volatile and does not provide a precise tool for short-term monitoring.

**Table 6:** Standard Error of the Rate of Growth; Concurrent Estimator  
(in percent points of annualized growth)

	Trend Component	Seasonally Adjusted Series
Monthly growth of the monthly series	3.56	3.38
Monthly growth of a centered 3-month moving average	2.60	2.83
Monthly growth of the last 3 months	1.99	1.79
Centered estimator of 12-month growth	1.61	1.64

Rates computed over longer periods will of course be more stable. Within the model-based approach, linearizing the rate, the standard error of the estimators can be computed in a similar manner to that of the rate  $R_t$ . Table 6 presents some examples: First, the second row contains the standard error of the estimator of the monthly rate of growth of a 3-month moving average. In order to minimize the phase effect, the moving average is centered, and hence the 1-period-ahead forecast of the component is included in its computation. The third row of table 6 presents the standard error of the estimator of the rate of growth of a one-sided 3-month moving average, formed by the last 3 periods. Finally, the last row contains the standard errors of a rate that measures the annual growth by comparing the 6-month-ahead forecast of the component with its value one year before this horizon. This rate, which smoothes the data over a 12-month period, is clearly the most stable one, and provides more sensible tolerance ranges ( $\pm 3$  p.p., approximately). The rate, however, can also be computed directly on the original series, without having to estimate the components; the standard error of the estimator when the series is used equals 1.65 p.p., slightly larger than the one obtained when the

trend is used.

These are some examples that illustrate inference in model-based unobserved component estimation procedures; additional references are Cleveland and Pierce (1981), Hillmer (1985), Burrige and Wallis (1985), and Maravall (1988a). What is worth stressing is the fact that the model-based approach provides the tools to apply, in a simple way, proper statistical inference to answer relevant questions involving unobserved components in time series.



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Fig.1 : HP FILTER (FOURIER TRANSFORM)

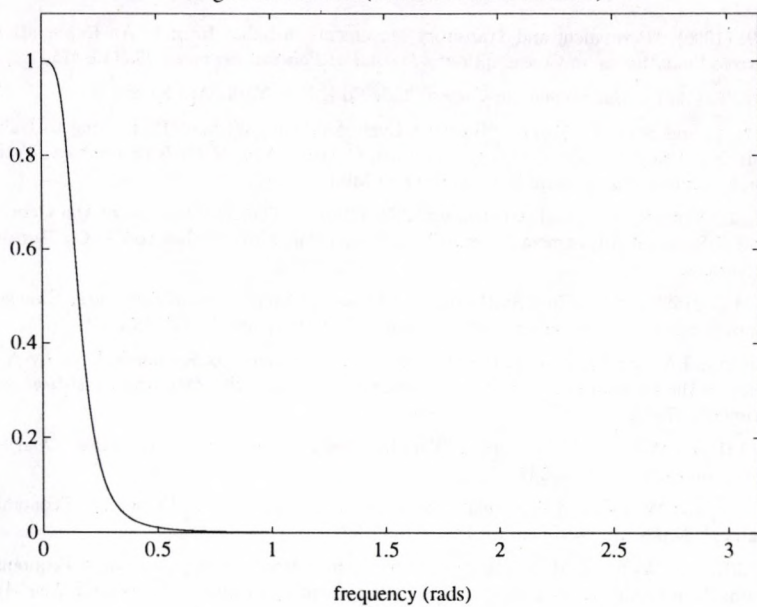


Fig.2 : X11 SEASONAL ADJUSTMENT FILTER (FOURIER TRANSFORM)

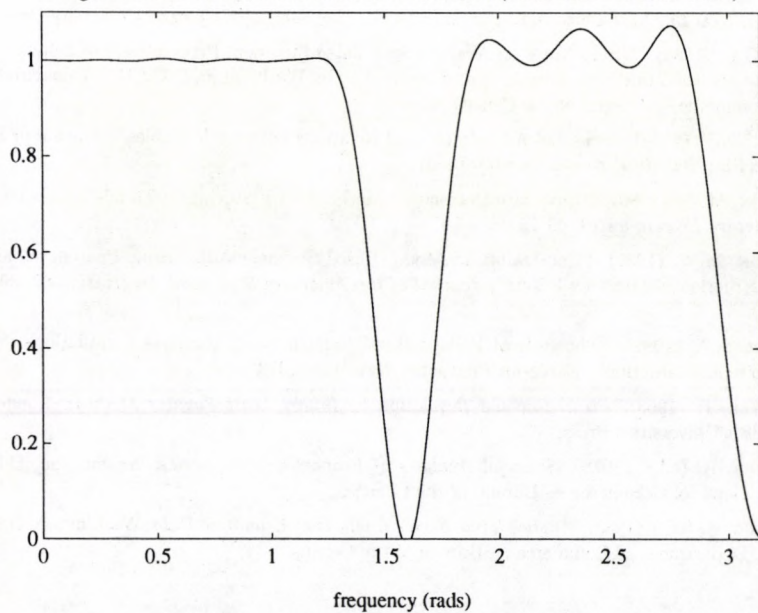


Fig.3 : HP TREND FOR  $t=70$ ; PRELIMINARY AND FINAL ESTIMATORS

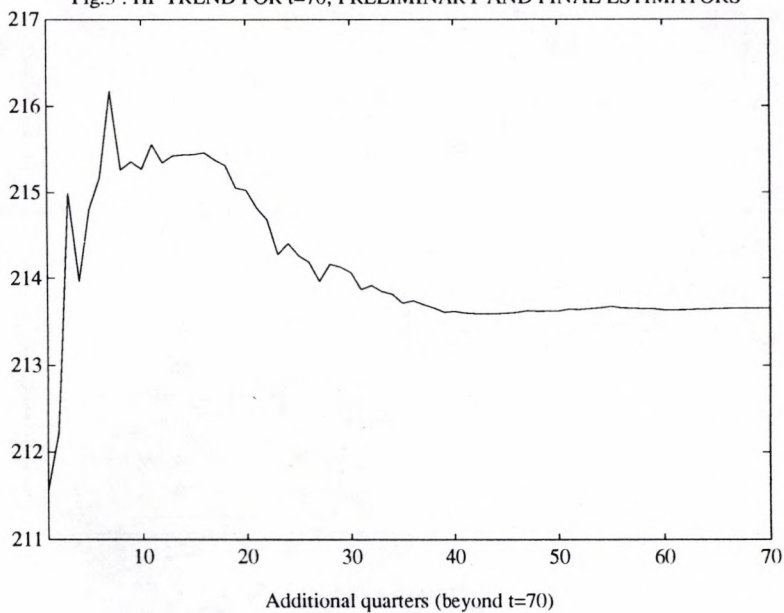


Fig.4 : DETRENDED WHITE-NOISE WITH THE HP FILTER

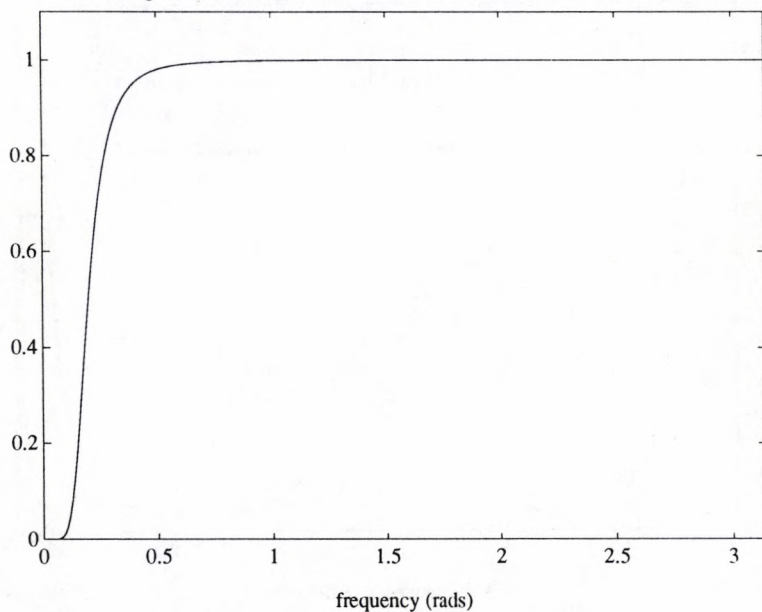




Fig.5 : SEASONAL COMPONENT EXTRACTED FROM WHITE-NOISE BY X11

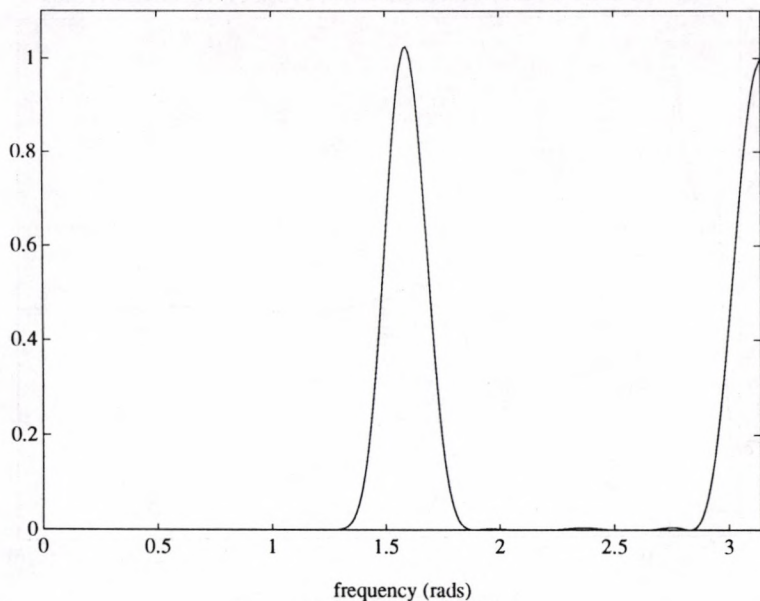


Fig.6 : TWO EXAMPLES WITH STABLE AND UNSTABLE COMPONENTS

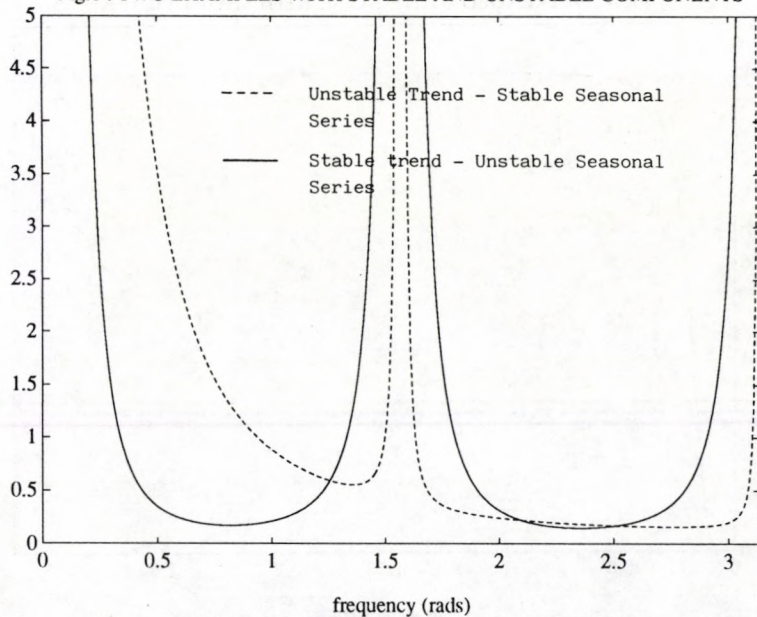


Fig.7a : UNSTABLE SEASONAL SERIES AND X11 SEASONAL COMPONENT

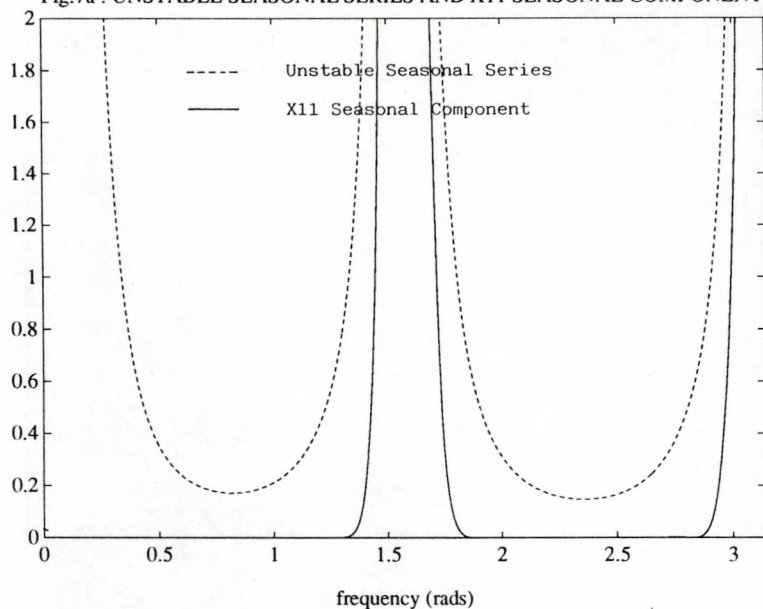


Fig.7b : SEASONALLY ADJUSTED SERIES

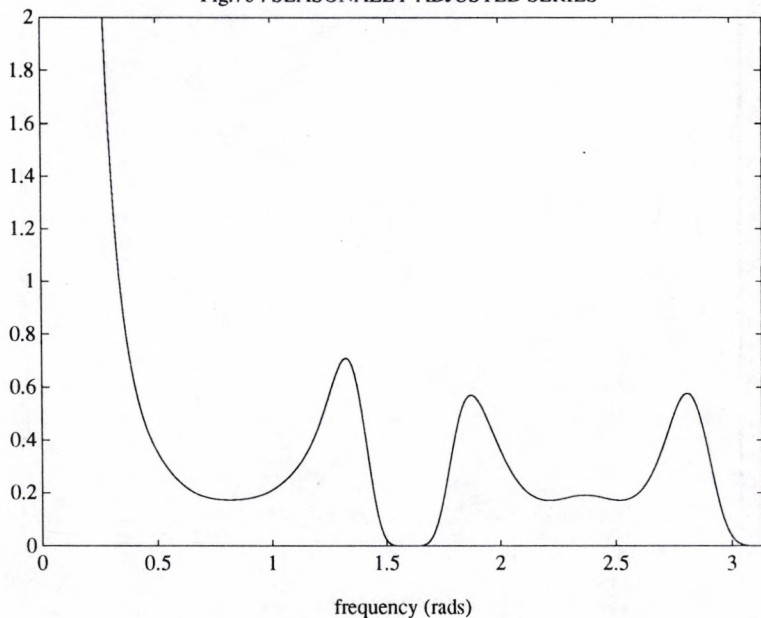




Fig.8a : UNSTABLE TREND SERIES AND HP TREND

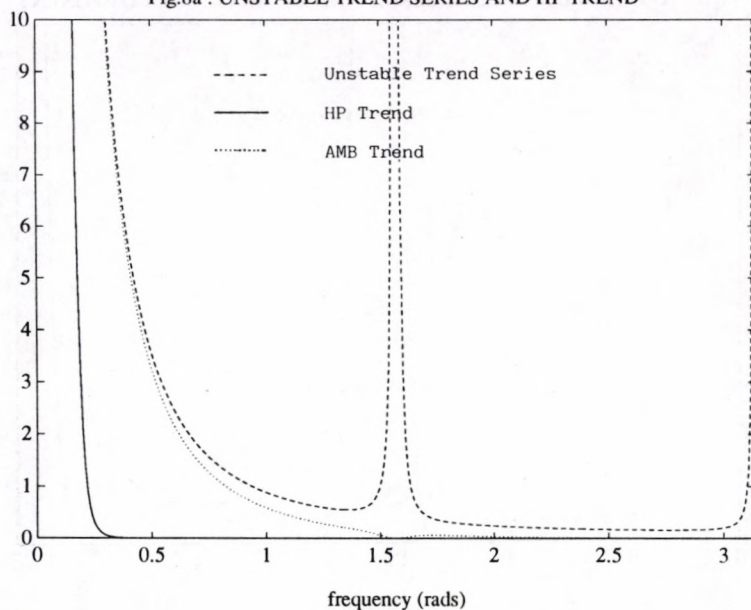


Fig.8b : DETRENDED SERIES

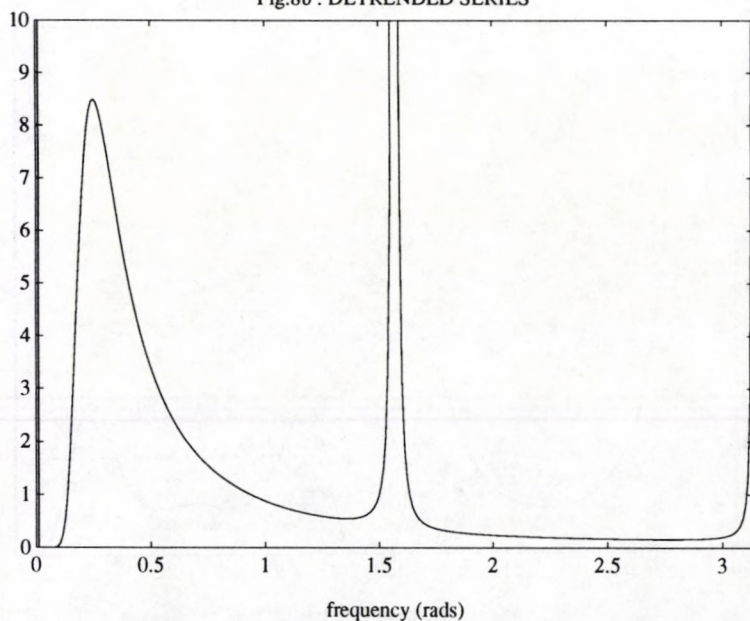


Fig. 9: TWO ADMISSIBLE DECOMPOSITION

Fig.9a : SERIES

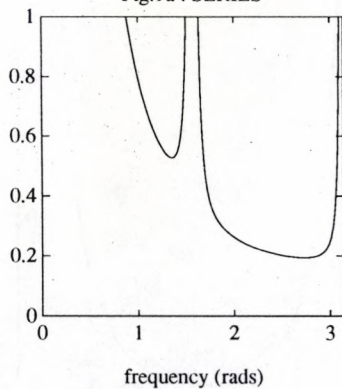


Fig.9b : SEASONAL COMP.

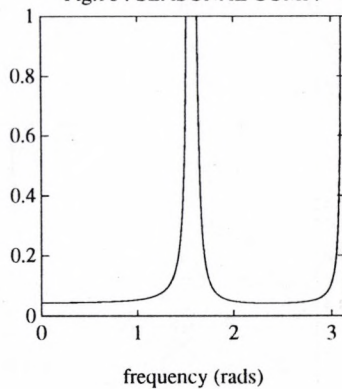


Fig.9c : TREND

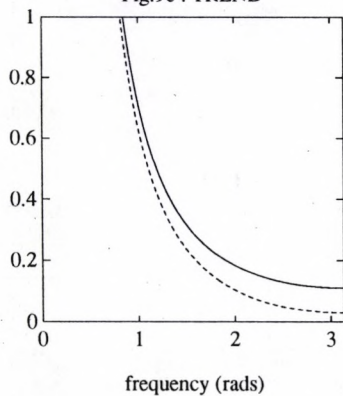


Fig.9d : IRREGULAR

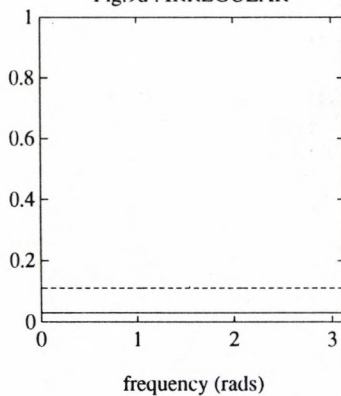




Fig.10a : UNSTABLE SEASONAL AND AMB SEASONAL COMPONENT

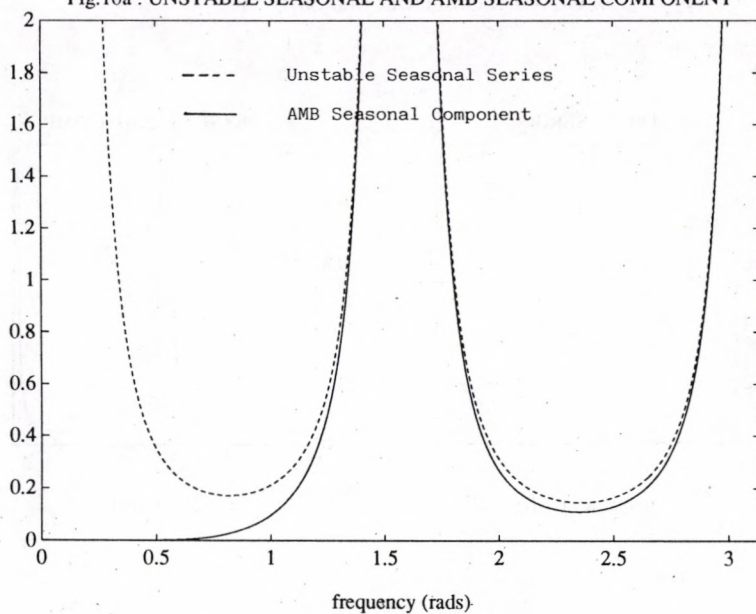
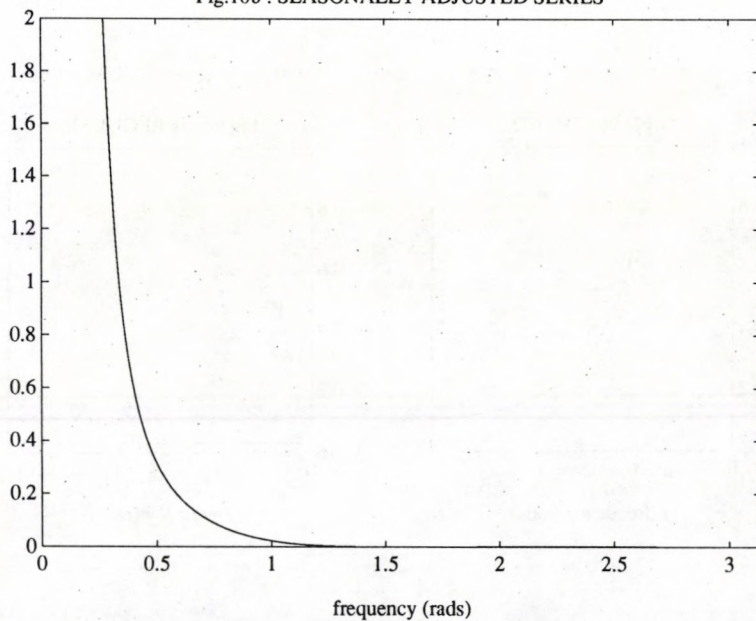


Fig.10b : SEASONALLY ADJUSTED SERIES



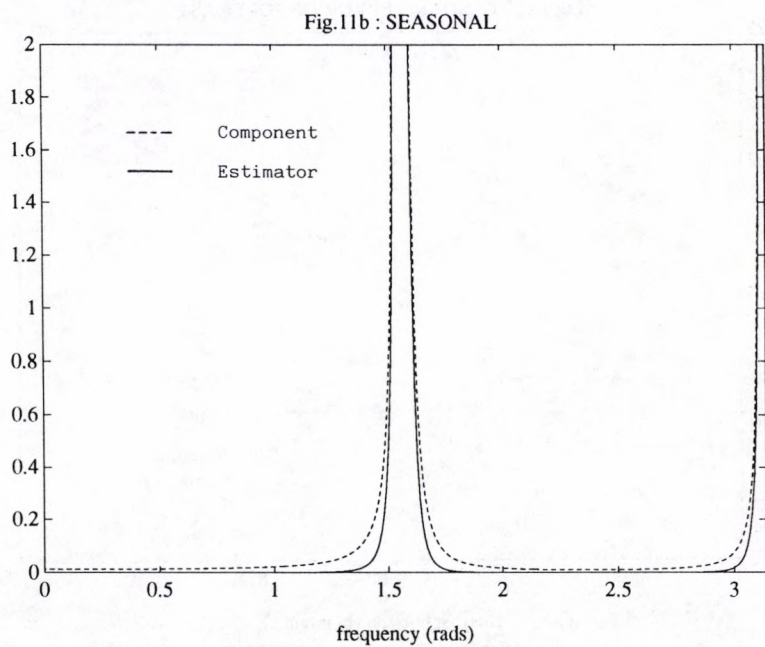
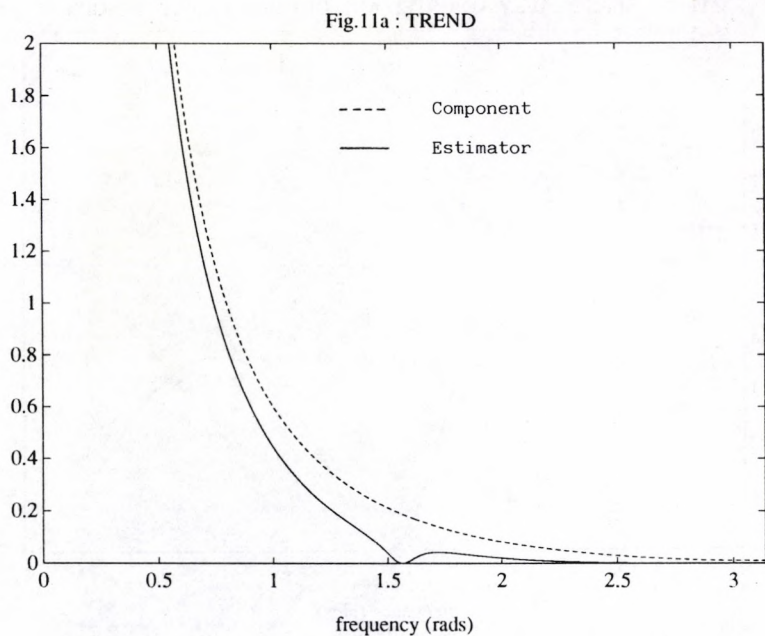




Fig.12 : SEASONALLY ADJUSTED AND DETRENDED WHITE-NOISE

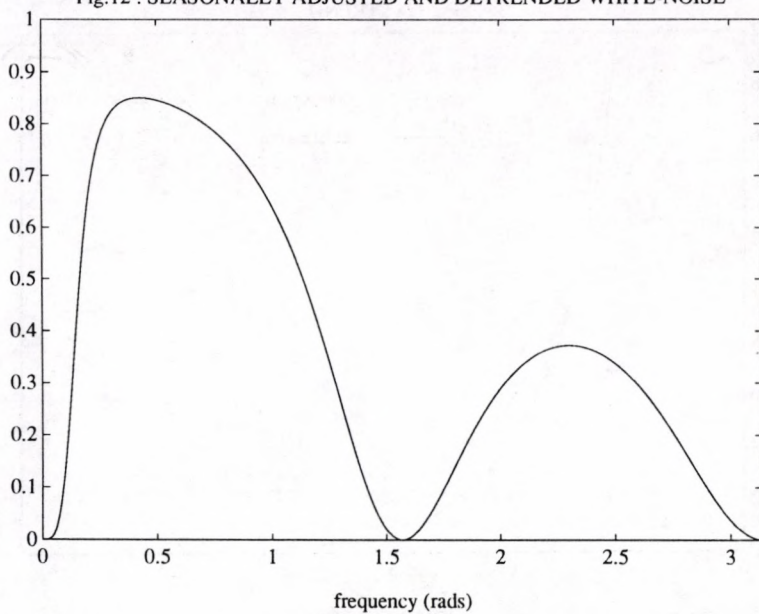
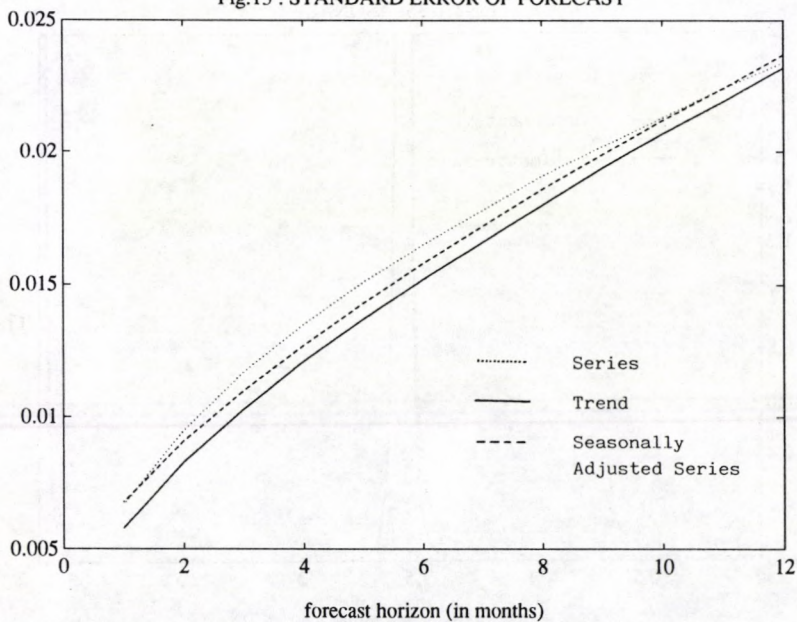


Fig.13 : STANDARD ERROR OF FORECAST





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